PRICING LYONS UNDER STOCHASTIC INTEREST RATES

Javier Fernández Navas
Instituto de Empresa
Dept. of Finance
María de Molina, 12, 4º
28006 Madrid, Spain
jfnavas@ie.edu

Abstract

In this study, one of the simplifying assumptions of the McConnell and Schwartz (1986) LYON pricing model is relaxed. We present a valuation model that incorporates stochastic interest rates. LYON prices are computed numerically. To ensure convergence, we use the modified explicit finite difference method of Hull and White (1990). For the Waste Management issue, we find that the value of the LYON is very sensitive to the market price of interest rate risk and somehow sensitive to the correlation coefficient between interest rates and stock returns.

Keywords
Convertible bonds, derivatives, LYONs, stochastic interest rates

Pricing LYONs under Stochastic Interest Rates

Javier F. Navas[*]

This Draft: January 2001
JEL Classification: C63, E43, G13
Keywords: convertible bonds, derivatives, LYONs, stochastic interest
rates

ABSTRACT

In this study, one of the simplifying assumptions of the McConnell and Schwartz (1986) LYON pricing model is relaxed. We present a valuation model that incorporates stochastic interest rates. LYON prices are computed numerically. To ensure convergence, we use the modified explicit finite difference method of Hull and White (1990). For the Waste Management issue, we find that the value of the LYON is very sensitive to the market price of interest rate risk and somehow sensitive to the correlation coefficient between interest rates and stock returns.

[*]Instituto de Empresa, Maria de Molina 12, Madrid 28006, Spain. Phone: +34 91 568-9600. Fax: +34 91 563-6859. E-mail: jfnavas@ie.edu.

I. Introduction

Liquid yield option notes, LYONs, are zero coupon convertible bonds that can be called by the issuer or redeemed by the investor at different prices through time. It is a proprietary product of Merrill Lynch, that created it in 1986. The complexity of the bond makes it more difficult to value and to sell, and the underwriter typically compensates this charging a higher underwriting fee (see Becker and Long (1997)).

This security has been priced, in a simple and practical way, by McConnell and Schwartz (1986) (MS hereafter). They assume that the value of a LYON depends on the issuer's stock price and that the interest rate is constant. These assumptions allow them to develop a model that captures most of the features of this security. However, as they point out, these assumptions may have an important impact on the pricing of LYONs.

The first assumption precludes the possibility of bankruptcy and overstates the value of the LYON, while the second assumption has a mixed effect. Uncertain interest rates would increase the value of the put and call features for the investor and the issuer respectively. Thus, by assuming constant interest rates, the model understates (due to the put feature) and overstates (due to the call feature) the value of the LYON. Consequently, the net effect of stochastic interest rates will depend on the characteristics of the LYON.

MS also assume that the conversion and redemption strategies followed by investors and the call strategy followed by the issuer are "optimal", i.e. investors follow conversion and redemption strategies that maximize the value of the LYON, and the issuer follows a call policy that minimizes the value of the LYON. However, the empirical evidence[1] seems to indicate that the call policies of most firms are not optimal.

---

[1] See Ingersoll (1977), for example.

In this paper, we develop a more sophisticated model for pricing LYONs, incorporating stochastic interest rates. We find that the value of the LYON can increase substantially with the market price of interest rate risk and the correlation between interest rates and stock returns.

This note is organized as follows. Next section presents the valuation model. In Section III, we apply the model to the pricing of a particular LYON issue. Finally, we conclude with Section IV.

II. The Model

For completeness, we first present the MS model. They assume that the value of the LYON, L, depends upon the issuer's stock price, S, which follows a geometric Brownian motion process. That is,

$$dS(t) = (S\mu_s - D(S,t)) \, dt + S\sigma_s dZ_s, \tag{1}$$

where μ_s is the drift of the stock price process, $D(S,t)$ is the total amount of dividends paid at time t, and σ_s is the instantaneous volatility of stock returns. MS use standard arbitrage arguments and show that the value of the LYON must satisfy the following partial differential equation (PDE)

$$\frac{1}{2}\sigma_s^2 S^2 L_{ss} + (Sr - D(S,t)) L_s + L_t - rL = 0, \tag{2}$$

where r is the instantaneous risk-free interest rate (assumed to be constant in the model) and the subscripts of L represents partial derivatives. The boundary conditions for this equation are given by the different features of the security:

- Maturity condition: $L(S, T) = \max(XS, F)$, where X is the conversion ratio (number of shares of the issuer's common stock into which the LYON can be converted), and F is the face value of the LYON.

- Conversion condition: $L(S, t) \geq XS$. This is true at any time, t, prior to or equal to maturity, T.

- Put (or redemption) condition: $L(S, t_p) \geq P(t_p)$, where t_p are the times at which the LYON can be redeemed, and $P(t_p)$ are the redemption prices.

- Call condition: $L(S, t) \leq \max(C(t), XS)$, where $C(t)$ is the call price of the LYON at time $t \leq T$.

Because of the complexity of these boundary conditions, no closed-form solution for the value of the LYON is known, and MS use numerical methods to solve equation (2) and compute LYON prices.

In this paper, we extend the MS model and assume that interest rates are stochastic. To avoid complexity, we suppose that the term structure of interest rates is given by the instantaneous interest rate, whose dynamics can be described by the following stochastic differential equation (SDE)

$$dr = \alpha(\mu_r - r)dt + r\sigma_r dZ_r. \tag{3}$$

Here, the interest rate converges to its long term mean, μ_r, at the velocity rate of α, and the conditional volatility of changes in the interest rate is proportional, with coefficient σ_r, to r.

Expressions (1) and (3) are related trough ρ, the instantaneous correlation coefficient between changes in interest rates and changes in stock returns; that is $dZ_s dZ_r = \rho dt$.

Thus we use the Brennan and Schwartz (1980) one-factor interest rate model, which belongs to the family of models given by $dr = \alpha(\mu_r - r)dt + r^\gamma \sigma_r dZ_r$. Our choice of

$\gamma = 1$ can be justified by the empirical evidence of Chan, Karolyi, Longstaff, and Sanders (1992), Uhrig and Walter (1996), and Navas (1999), whose estimates of γ are 0.77, 1.50, and 1.77 for the German, U.S., and Spanish market, respectively.

We now assume that the value of the LYON depends upon the issuer's stock price (which follows the SDE (1)), and the instantaneous interest rate (which follows the SDE (3). Using no arbitrage arguments, it is easy to derive the following PDE for the value of the LYON

$$
\frac{1}{2}\sigma_s^2 S^2 L_{ss} + \frac{1}{2}\sigma_r^2 r^2 L_{rr} + rS\rho\sigma_r\sigma_s L_{rs} + (Sr - D(S,t))L_s
$$
$$
+ \left(\alpha(\mu_r - r) - \lambda\sigma_r r\right)L_r + L_t - rL = 0, \tag{4}
$$

where λ represents the market price of interest rate risk.

To compute LYON prices, we solve equation (4) numerically, subject to the corresponding boundary conditions. We use the explicit finite difference method because its simplicity and efficiency (see Geske and Shastri (1985)). To ensure that the computed values converge to the solution of the PDE (4), we implement the modified explicit finite difference method of Hull and White (1990), extended to the case of two state variables (the stock price and the interest rate).

III. An Application

We study the case of one of the first LYON issues: the Waste Management issue on April 12, 1985. For a detailed summary of the characteristics of this particular product, see Table 1.

We first use the Hull and White (1990) method to solve the PDE (2) for the case of constant interest rates. We assume that dividend payments are given by $D(S,t) = yS$,

where y is the dividend yield of the stock. Table II shows that the MS model solved with the explicit finite difference method of Hull and White (1990) produces LYON prices (column 5) very similar to those reported by MS (column 4). The model overprices the Waste Management LYON by about $5 or 2%. Figures 1 through 3 present a sensitivity analysis of theoretical LYON values to changes in the issuer's stock price, stock price volatility, dividend yield, and interest rate[2]. We observe that the value of the LYON is not very sensitive to changes in the dividend yield, somehow sensitive to changes in the stock price volatility, and very sensitive to changes in the interest rate (when the stock price is $46, as the interest rate decreases from 0.1121 to 0.0721, the LYON price increases by 15.1%).

We next apply the stochastic interest rate (SIR hereafter) model to the Waste Management LYON. For the stock price process we use the parameters of the MS model, that is $\sigma_s = 0.30$ and $y = 0.016$. For the interest rate process we take the parameters from Chan, Karolyi, Longstaff, and Sanders (1992). They use one-month U.S. Treasury bills from June, 1964 to December, 1989 to estimate the parameters of the Brennan and Schwartz (1980) model. Consequently, we have that $\alpha = 0.3142$, $\mu_r = 0.0770$, and $\sigma_r = 0.3442$. To be consistent with MS, we take as current interest rate[3] $r = 0.1121$.

Table III compares the stochastic and constant interest rate models. We observe that the SIR model produces a richer set of LYON prices than the MS model. For the former model we consider six cases, in which the market price of interest rate risk (λ) takes the values -0.5 and 0.0, while the correlation coefficient (ρ) takes the values[4] -0.2, 0.0, and 0.2. We see that LYON prices increase with λ and ρ. When $\lambda = -0.5$ and $\rho = -0.2$, the SIR model gives LYON prices that are lower than those of the MS model.

---

[2]Note that our LYON prices are very close to the MS prices in all instances except one. When the interest rate is 0.0721, the LYON prices that we obtain are approximately a 4% lower than those calculated by MS. The cause of this discrepancy remains an open question.

[3]In our model, this rate should be the one-month Treasury bill yield on April 12, 1985. Using the Fama (1984) data set, this rate is 0.0771 on April 30, 1985 (in annualized form). However MS use the yield of intermediate term bonds of the same risk as the Waste Management bond.

[4]In practice, the correlation coefficient is small. For instance, Brennan and Schwartz (1980) find that $\rho = -0.01$ for a sample of Treasury Bills and the CRSP market index.

If we increase ρ to 0.0, we obtain prices similar in both models. When $\rho = 0.2$, the SIR model gives prices higher than the MS model. When the market price of risk is zero, the SIR model overprices the LYON (relative to the MS model) for the three correlation coefficients used. For example, when the stock price is \$49 and $\rho = 0.0$, the SIR model price is 6.9% higher than the MS price.

These results can be partially explained by the effect of λ on the shape of the initial yield curve. Figure 4 shows the term structure on interest rates implied by the Brennan and Schwartz (1980) model for different values of λ. The curves have been obtained numerically, computing the yield to maturity of a set of discount bonds. When the pure expectations theory of the term structure holds and $\lambda = 0$, the yield curve starts at 0.1121 and falls fairly quickly to 0.077. As a consequence, LYON prices are higher than in the MS model, even for low correlation coefficients. When $\lambda = -0.5$, the yield curve starts at 0.1121 and rises slowly (the 20-year interest rate is under 0.13). Thus LYON prices are lower than in the MS model, except for high correlation coefficients.

IV. Final Remarks

An important issue when pricing a bond (convertible or otherwise) is to match the initial yield curve. Using a simple one-factor interest rate model, this curve can be partially approximated. The assumption of constant interest rates can significantly affect theoretical LYON prices. MS find that their model overprices the Waste Management LYON on its first month of life. We show that decreasing the market price of risk or the correlation coefficient between interest rates and stock returns, we can obtain LYON prices consistent with market data. Of course, the same results could possibly be obtained with a model that allows for the possibility of bankruptcy of the issuing firm.

References

Becker, Michael W., and Michael S. Long, 1997, An Explanation of Underwriting Spread Differentials on Complex Securities, *Financial Management* 26, 35–43.

Brennan, Michael J., and Eduardo S. Schwartz, 1980, Analyzing Convertible Bonds, *Journal of Financial and Quantitative Analysis* 15, 907–929.

Chan, K. C., G. Andrew Karolyi, Francis A. Longstaff, and Anthony B. Sanders, 1992, An Empirical Comparison of Alternative Models of the Short-Term Interest Rate, *The Journal of Finance* 47, 1209–1227.

Fama, Eugene F., 1984, Term Premiums in Bond Returns, *Journal of Financial Economics* 13, 529–546.

Geske, R., and K. Shastri, 1985, Valuation of Approximation: a Comparison of Alternative Approaches, *Journal of Financial and Quantitative Analysis* 20, 45–72.

Hull, John, and Alan White, 1990, Valuing Derivative Securities using the Explicit Finite Difference Method, *Journal of Financial and Quantitative Analysis* 25, 87–100.

Ingersoll, Jonathan E., 1977, An Examination of Corporate Call Policies on Convertible Securities, *Journal of Financial Economics* 32, 463–478.

McConnell, John J., and Eduardo Schwartz, 1986, LYON Taming, *The Journal of Finance* 41, 561–576.

Navas, Javier F., 1999, Consistent versus Non-Consistent Term Structure Models: Some Evidence from the Spanish Market, *The Journal of Fixed Income* 9, 42–60.

Uhrig, Marliese, and Ulrich Walter, 1996, A New Numerical Approach for Fitting the Initial Yiedl-Curve, *Journal of Fixed Income* 5, 82–90.

Table I

**Put and call exercise prices and dates for the Waste Management, Inc.
Lyon issue on April 12, 1985.**

Date	Put price	Call price
12/04/1985	–	272.50
06/30/1986	–	297.83
06/30/1987	–	321.13
06/30/1988	301.87	346.77
06/30/1989	333.51	374.99
06/30/1990	375.58	406.00
06/30/1991	431.08	440.08
06/30/1992	470.75	477.50
06/30/1993	514.07	518.57
06/30/1994	561.38	563.63
06/30/1995	613.04	613.04
06/30/1996	669.45	669.45
06/30/1997	731.06	731.04
06/30/1998	798.34	798.34
06/30/1999	871.80	871.80
06/30/2000	952.03	952.03
01/21/2001	–	1000.00

The face value of each lyon is \$1000, the maturity is January 21, 2001, the conversion rate is 4.36 shares of stock per bond. Waste Management may not call the bond prior to June 30, 1987, unless the price of their common stock rises above \$86.01 (the stock price on April 12, 1985 was \$52.25). After that date, Waste Management can call the bond at any time before than or at maturity. If the lyon is called between the dates shown in the table, the call price is adjusted (at a rate of 9% per year compounded semiannually) to reflect the interest accrued since the immediately preceding call date. The investor can elect to put the bond to Waste Management only on the dates given in the table.

Table II

Waste Management LYON prices for the constant interest rate case

Date	Stock price	Lyon market price	MS	MS(HW)
04/12/1985	52.250	258.75	262.7	262.9
04/15/1985	53.000	258.75	264.6	264.8
04/16/1985	52.625	257.50	263.7	263.8
04/17/1985	52.000	–	262.1	262.2
04/18/1985	52.365	257.50	263.0	263.2
04/19/1985	52.750	257.50	264.0	264.1
04/22/1985	52.500	257.50	263.3	263.4
04/23/1985	53.250	260.00	265.3	265.3
04/24/1985	54.250	265.00	267.9	267.9
04/25/1985	54.250	265.00	267.9	267.9
04/26/1985	54.000	265.00	267.2	267.4
04/29/1985	53.750	260.00	266.6	266.7
04/30/1985	52.125	260.00	262.4	262.6
05/01/1985	49.750	252.50	256.7	256.8
05/02/1985	50.500	250.00	258.4	258.6
05/03/1985	50.750	252.50	259.0	259.2
05/06/1985	50.500	252.50	258.4	258.6
05/07/1985	50.875	255.00	259.3	259.6
05/08/1985	50.750	253.75	259.0	259.2
05/09/1985	51.250	255.00	260.3	260.4
05/10/1985	53.125	260.00	265.0	265.1

MS represents the LYON prices reported by McConnell and Schwartz (1986), while MS(HW) represents the LYON prices when the McConnell and Schwartz (1986) model is solved using the explicit finite difference method of Hull and White (1990). The parameters of the model are dividend yield $y = 0.016$, stock price volatility $\sigma_s = 0.30$, and current interest rate $r = 0.1121$.

9

Table III

Waste Management LYON prices for the stochastic interest rate case

			λ					
			-0.5			0.0		
Stock	Actual lyon		ρ			ρ		
price	price	MS(HW)	-0.2	0.0	0.2	-0.2	0.0	0.2
49	–	255.08	251.45	253.84	256.16	268.73	270.64	272.63
50	–	257.44	253.84	256.17	258.57	270.55	272.54	274.50
51	–	259.73	256.28	258.76	261.03	272.45	274.48	276.41
52	–	262.20	258.82	261.15	263.54	274.39	276.33	278.42
53	258.75	264.82	261.39	263.82	266.13	276.41	278.57	280.42
54	265.00	267.41	264.07	266.44	268.77	278.53	280.45	282.55
55	–	269.91	266.79	269.08	271.43	280.70	282.91	284.70

MS(HW) represents the McConnell and Schwartz (1986) constant interest rate model with the parameters reported in Table 1, implemented using the explicit finite difference method of Hull and White (1990). Columns 4 to 9 provide LYON prices for the SIR model for different market prices of risk and correlation coefficients, with the parameters $\alpha = 0.3142$, $\mu_r = 0.077$, and $\sigma_r = 0.3442$.
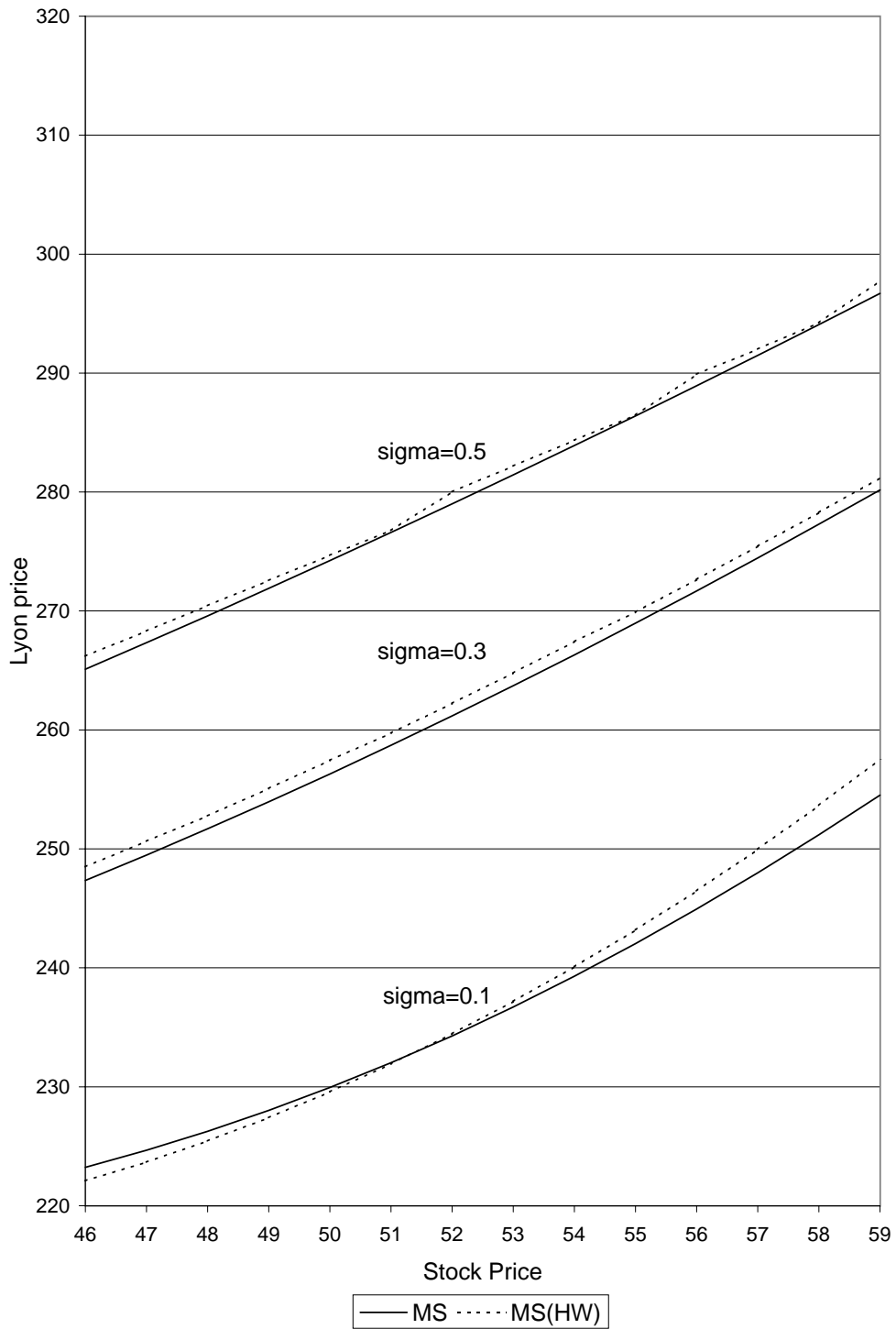
Figure 1. Sensitivity of LYON values in the constant interest rate case for changes in the stock price and the stock price volatility.
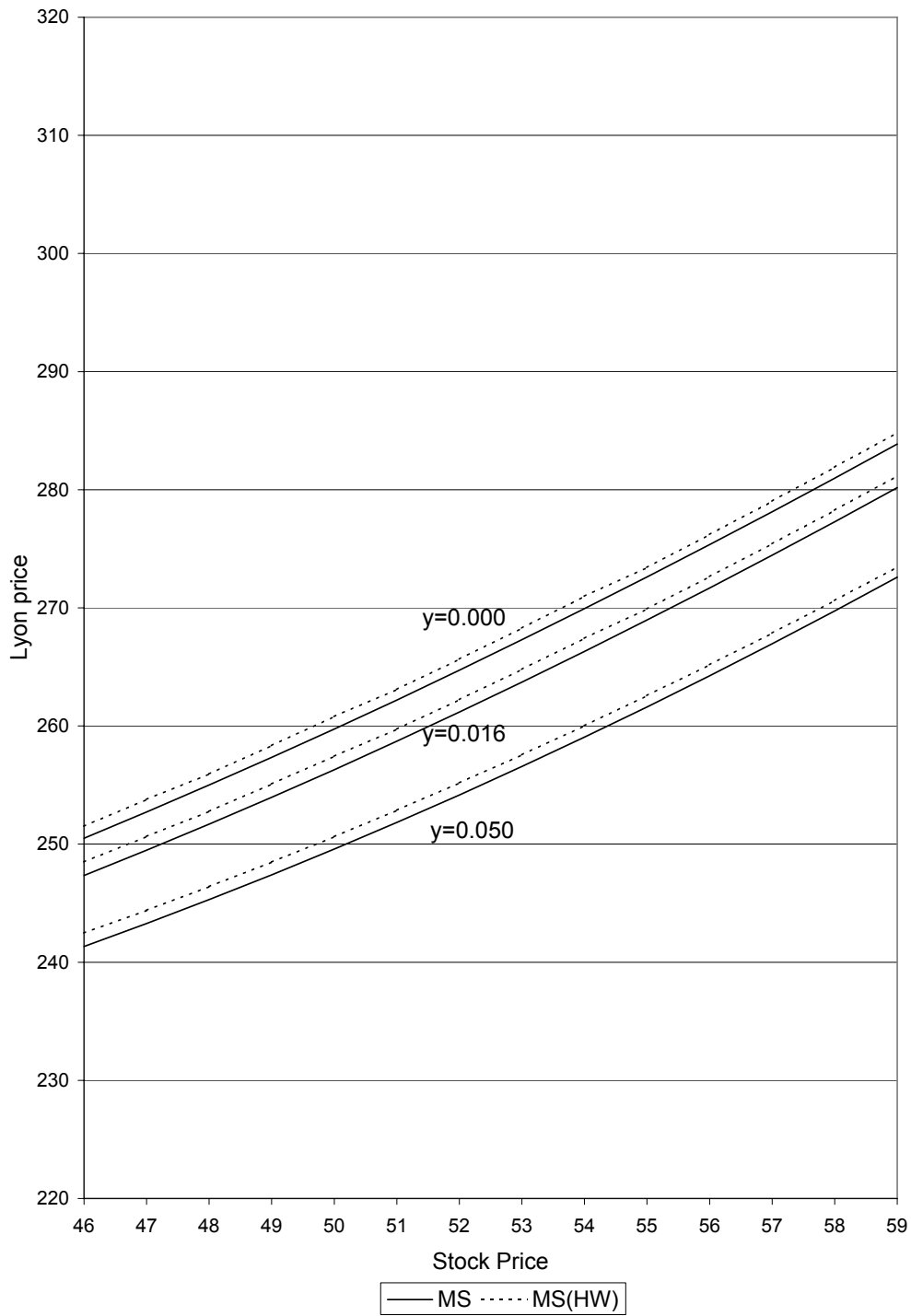
Figure 2. Sensitivity of LYON values in the constant interest rate case for changes in the stock price and the dividend yield.
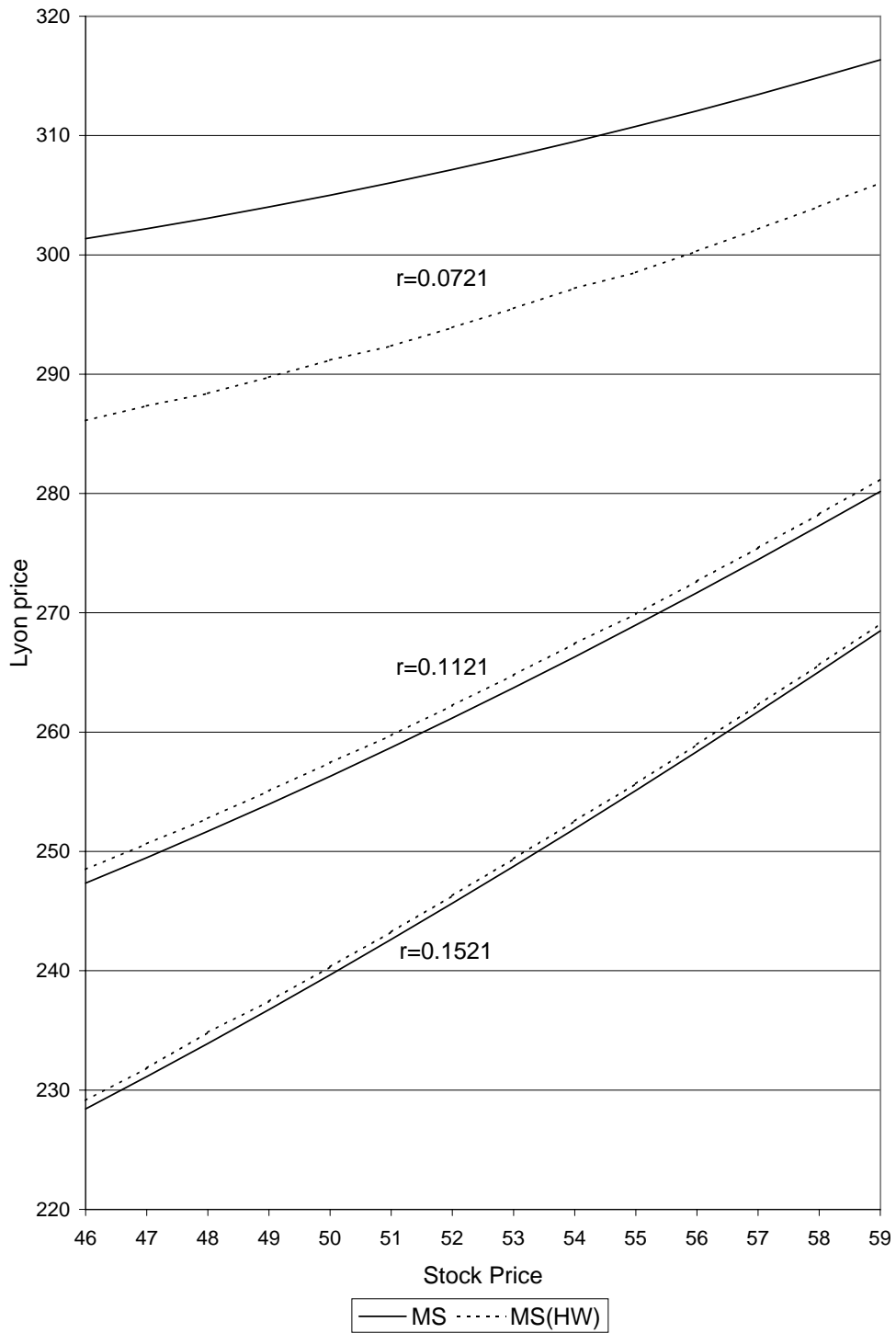
Figure 3. Sensitivity of LYON values in the constant interest rate case for changes in the stock price and the interest rate.
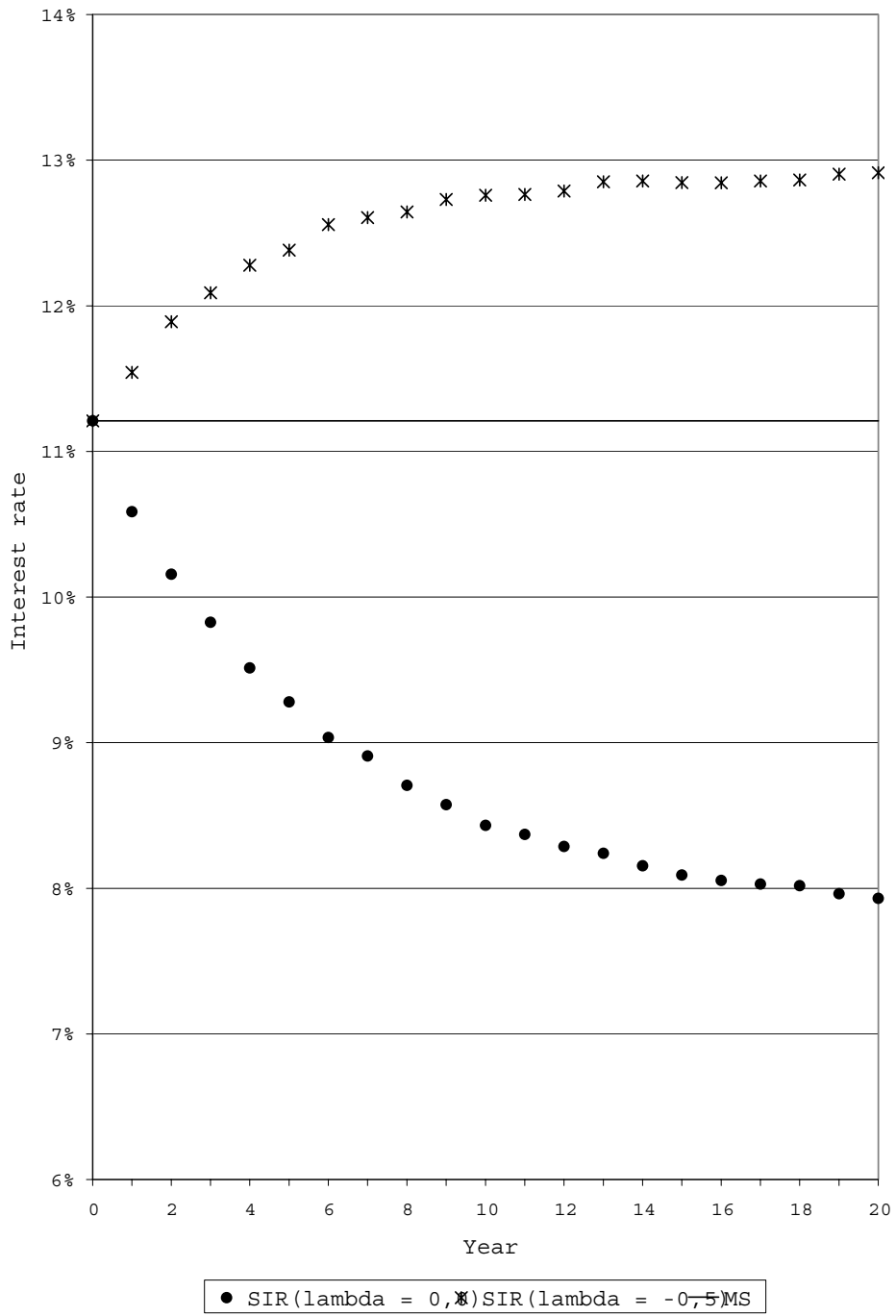
13

Figure 4. Implied term structure of interest rates.