IE Working Paper **DF8-103-I** **15/02/2002**

ON JUMP-DIFFUSION PROCESSES FOR ASSET RETURNS

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Abstract

A common way to incorporate discontinuities in asset returns is to add a Poisson process to a Brownian motion. The jump-diffusion process provides probability distributions that typically fit market data better than those of the simple diffusion process. To compare the performance of these models in option pricing, the total volatility of the jump-diffusion process must be used in the Black-Scholes formula. A number of authors, including Merton (1976a & b), Ball and Torous (1985), Jorion (1988), and Amin (1993), miscalculate this volatility. We show that if an investor uses Merton's volatility rate in the Black-Scholes (1973) model, she will underprice (overprice) some options, relative to the jump-diffusion model of Merton (1976a). However, if she used the correct volatility, she would overprice (underprice) the same options. We also show that the price difference between these models can be larger for some options and smaller for others than what was previously reported.

Keywords
Jump-diffusion process, option pricing, volatility smile

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This Draft: July 2001

JEL Classification: G13

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Abstract

A common way to incorporate discontinuities in asset returns is to add a Poisson process to a Brownian motion. The jump-diffusion process provides probability distributions that typically fit market data better than those of the simple diffusion process. To compare the performance of these models in option pricing, the total volatility of the jump-diffusion process must be used in the Black-Scholes formula. A number of authors, including Merton (1976a & b), Ball and Torous (1985), Jorion (1988), and Amin (1993), miscalculate this volatility. We show that if an investor uses Merton's volatility rate in the Black and Scholes (1973) model, she will underprice (overprice) some options, relative to the jump-diffusion model of Merton (1976b). However, if she used the correct volatility, she would overprice (underprice) the same options. We also show that the price difference between these models can be larger for some options and smaller for others than what was previously reported.

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The diffusion process is widely used to describe the evolution of asset returns over time. In option pricing it allows the use of Black and Scholes-type formulae to value European options on stocks, foreign currencies, interest rates, commodities and futures. Under this process, instantaneous asset returns are normally distributed. However, the distributions observed in the market exhibit non-zero skewness and higher kurtosis than the normal distribution[1], which produces pricing errors when the Black-Scholes formula is used.

A way to obtain distributions consistent with market data is to assume that the stock price follows a jump-diffusion process. Merton (1976b) develops a model in which the arrival of normal information is modeled as a diffusion process, while the arrival of abnormal information is modeled as a Poisson process. The jump-diffusion process can potentially describe stock prices more accurately at the cost of making the market incomplete, since jumps in the stock price cannot be hedged using traded securities. If the market is incomplete, the payoffs of the option cannot be replicated, and the option cannot be priced. To overcome this problem, Merton assumes that the jump risk is diversifiable and, consequently, not priced in equilibrium. He then derives a closed-form expression for the price of a call option.

To study the performance of the jump-diffusion model, Merton (1976a) compares option prices computed with his model with those obtained with the Black-Scholes formula. He concludes that there can be significant differences in option prices for deep out-of-the-money (DOTM) options and short maturity options when there are large and infrequent jumps. However, for this comparison to be meaningful, the total volatility of the jump-diffusion process (ν) must be used in the Black-Scholes formula. Instead, Merton (1976a & b) uses a "volatility rate" (ν_M) that is smaller than ν.

Similarly, Ball and Torous (1985) analyze a sample of NYSE listed common stocks

[1]See, for example, Fama (1965), Jorion (1988), Hsieh (1988), Bates (1996), and Campa, Chang, and Rieder (1997).

and find the existence of lognormal jumps in most of the daily returns considered. They then study Merton and Black-Scholes call option prices and find small differences. However, rather than using the total volatility of the process, they follow Merton (1976a & b) and use the same volatility rate.

Other examples where ν_M is used are Jorion (1988), who adapts Merton's model to price foreign currency options, and Amin (1993), who develops a discrete-time option pricing model under a jump-diffusion process.

Despite the empirical evidence, most analysts routinely use the Black-Scholes model to price options. As mentioned before, Merton (1976a) shows that the Black-Scholes formula can substantially undervalue DOTM options when there are jumps in stock prices. As these options become less DOTM, the underpricing turns quickly into overpricing Hence, it is important for practitioners to know the critical stock prices where these shifts occur.

In this paper we determine these crossover points using the total volatility of the jump-diffusion process. We show that some options undervalued with ν_M are in fact overvalued and vice versa. We also find that, generally, the difference in option prices given by the Black-Scholes and Merton models is greater for near-the-money options (NTM) and smaller for DOTM and deep in-the-money (DITM) options than what was previously reported .

The rest of the paper is organized as follows. Next section briefly reviews the Merton (1976b) model and presents the total variance of the jump-diffusion process. Using this variance, we study in Section 2 the pricing error of an investor who uses the Black-Scholes formula to price options when the stock price follows a jump-diffusion process. We compare this error with the error previously reported in the literature. Finally, we conclude with Section 3.

1 Merton's Jump-Diffusion Formula

We consider a continuous trading economy with trading interval $[0, \tau]$ for a fixed $\tau > 0$, in which there are three sources of uncertainty, represented by a standard Brownian motion $\{Z(t) : t \in [0, \tau]\}$ and a independent Poisson process $\{N(t) : t \in [0, \tau]\}$ with constant intensity λ and random jump size Y on the filtered probability space $(\Omega, \mathcal{F}, \mathrm{Q}, \{\mathcal{F}_\mathrm{t} : \mathrm{t} \in [0, \tau]\})$.

We define $\mathcal{F}_\mathrm{t} \equiv \mathcal{F}_\mathrm{t}^\mathrm{Z} \vee \mathcal{F}_\mathrm{t}^\mathrm{N}$ and $\mathcal{F} \equiv \mathcal{F}_\tau$, where $\mathcal{F}_\mathrm{t}^\mathrm{Z}$ and $\mathcal{F}_\mathrm{t}^\mathrm{N}$ are the smallest right-continuous complete σ-algebras generated by $\{Z(s) : s \leq t\}$ and $\{N(s) : s \leq t\}$ respectively.

Merton (1976b) assumes that the stock price dynamics are described by the following stochastic differential equation

$$\frac{dS(t)}{S(t-)} = (\alpha - \lambda\kappa)\, dt + \sigma dZ(t) + (Y(t) - 1)\, dN(t), \tag{1}$$

where $\kappa \equiv E\{Y(t) - 1\}$ is the expected relative jump of $S(t)$.

Assuming that jump sizes are lognormally distributed with parameters μ and δ, and that the jump risk is diversifiable, Merton shows that the option price is given by

$$F(S(t), \tau, E, \sigma^2, r; \mu, \delta^2, \lambda) = \sum_{n=0}^{\infty} \frac{e^{-\lambda'\tau}(\lambda'\tau)^n}{n!} W(S(t), \tau, E, \sigma_n^2, r_n) \tag{2}$$

where $W(S(t), \tau, E, \sigma_n^2, r_n)$ is the Black-Scholes option price, and

$$\begin{aligned} r_n &\equiv r + \frac{n}{\tau}\left(\mu + \frac{\delta^2}{2}\right) - \lambda\kappa, \\ \sigma_n^2 &\equiv \sigma^2 + \frac{n}{\tau}\delta^2, \\ \lambda' &\equiv \lambda(1+\kappa) = \lambda \exp\left(\mu + \frac{\delta^2}{2}\right). \end{aligned}$$

Unlike in the diffusion case, when changes in stock prices are given by expression (1) the instantaneous stock returns are not normally distributed. The distribution will have non-zero skewness and will exhibit leptokurtosis when compared to a Gaussian distribution, which is consistent with the empirical evidence. There will not be, in general, a closed-form expression for the density function of the distribution, but we can easily compute the moments of stock returns. Applying Itô's formula to (1) we have that, under Q, the expected natural logarithm of the stock price is given by

$$\begin{aligned} E\left\{\ln\frac{\mathrm{S(t)}}{\mathrm{S(0)}}\right\} &= \left(\alpha - \lambda\kappa - \frac{\sigma^2}{2}\right)t + E\left\{E\left\{\mathrm{lnY(n(t))}|\mathcal{F}_{\mathrm{T}}^{\mathrm{N}}\right\}\right\} \\ &= \left(\alpha - \lambda\kappa - \frac{\sigma^2}{2}\right)t + E\left\{n(t)\mu\right\}, \\ &= \left(\alpha - \lambda\kappa - \frac{\sigma^2}{2}\right)t + \lambda t\mu, \end{aligned}$$

where $Y(n) \equiv \prod_{i=1}^{n} Y_i$, $\{Y_i, i = 1, 2, \ldots, n\}$ are the random jump amplitudes, and $n(t)$ is a Poisson random variable with parameter λt.

Since the Brownian motion and the Poisson process are independent by construction (see Protter (1995)), to calculate the variance, we can write

$$Var\left\{\ln\frac{\mathrm{S(t)}}{\mathrm{S(0)}}\right\} = Var\left\{\sigma Z(t)\right\} + Var\left\{\mathrm{lnY(n(t))}\right\} \tag{3}$$

Notice that $\sum_{i=1}^{n} \mathrm{lnY_i}$ is conditionally distributed as a Gaussian with mean $n\mu$ and variance $n\delta^2$. However, this does not imply that $Var\left\{\mathrm{lnY(n(t))}\right\} = \lambda t\delta^2$ since now $n(t)$ is random. Using the fact that for any random variable $u, Var\{u\} = -E\{u\}^2 + E\left\{u^2\right\}$ we have that

$$\begin{aligned} Var\left\{\mathrm{lnY(n(t))}\right\} &= -E\left\{\mathrm{lnY(n(t))}\right\}^2 + E\left\{\left[\mathrm{lnY(n(t))}\right]^2\right\} \\ &= -(\lambda t\mu)^2 + E\left\{E\left\{\left[\mathrm{lnY(n(t))}\right]^2|\mathcal{F}_{\mathrm{T}}^{\mathrm{N}}\right\}\right\} \end{aligned}$$

$$
\begin{aligned}
&= -(\lambda t\mu)^2 + E\left\{n(t)\delta^2 + n(t)^2\mu^2\right\} \\
&= -(\lambda t\mu)^2 + \delta^2\lambda t + \mu^2\left(\lambda t + \lambda^2 t^2\right) \\
&= \lambda\left(\mu^2 + \delta^2\right)t. \qquad (4)
\end{aligned}
$$

Hence, the total variance of the natural logarithm of the stock price under a jump-diffusion process is given by

$$
Var\left\{\ln\frac{\mathrm{S(t)}}{\mathrm{S(0)}}\right\} = \left(\sigma^2 + \lambda\left(\mu^2 + \delta^2\right)\right)t. \qquad (5)
$$

A similar result can be found in Press (1967), Navas (1994), and Das and Sundaram (1999). Merton (1976a & b), Ball and Torous (1985), Jorion (1988), and Amin (1993) miscalculate this variance as $(\sigma^2+\lambda\delta^2)t$, which will only be correct when $\mu = 0$. However, they assume that the expected jump size is zero, i.e. $E\{Y-1\} = 0$, which is equivalent to take $\mu = -\frac{\delta^2}{2}$. Obviously, when δ^2 increases the error in the variance will increase.

2 Implications for Option Pricing

If an investor prices options according to the Black-Scholes model but "the market" prices options according to Merton's jump-diffusion process, she will underprice out-of-the-money and in-the-money options and she will overprice at-the-money (ATM) options, since that the true stock return distribution will have fatter tails than the Gaussian distribution. Hence, if the investor estimates the volatility parameter of the diffusion process implicitly from market prices, she will observe the well known volatility smile. In Figure 1 we price a call option with different exercise prices with the Merton (1976b) model using the parameters $\tau = 0.5, \sigma^2 = 0.05, r = 0.10, \mu = -0.025, \delta^2 = 0.05$, and $\lambda = 1$. Then, we take those prices as market prices and compute implied volatilities for the Black-Scholes formula. We observe that in order to fit "market" prices correctly, the volatility of the diffusion process in the Black-Scholes model must be set higher for out-of-the-money (OTM) and ITM options than for ATM options. The curve is not symmetric because of the presence of skewness in the distribution of stock returns.

Insert Figure 1 about here.

As an example of the potential pricing "error" that our investor will face, we replicate in Table 1 part of Table IV of Ball and Torous (1985), using ν^2 instead of ν_M^2 (recall that the total variance of the process per unit of time, ν^2, is $\sigma^2 + \lambda\,(\mu^2 + \delta^2)$, while ν_M^2 is defined as $\sigma^2 + \lambda\delta^2$). In the table we compare Merton and Black-Scholes prices for different parameters of the jump-diffusion process. We value a 6-month stock call option with exercise price 35 when the stock is trading at 38. The variance of the diffusion part is $\sigma^2 = 0.05$, and the interest rate is $r = 0.10$.

Insert Table 1 about here.

We first follow Merton (1976a & b) and Ball and Torous (1985) and study the case where the expected relative jump size of the stock (k) is 0, i.e. we take $\mu = -\frac{\delta^2}{2}$. To be consistent with their notation, we represent Merton prices by F and the investor appraised option value using Black-Scholes formula by F_e, that is $F \equiv F(S(t), \tau, E, \sigma^2, r; \mu, \delta^2, \lambda)$ and $F_e(\nu^2) \equiv W(S(t), \tau, E, \nu^2, r)$.

In the second row of the table, we assume that the variance of the log of the jump size is equal to the variance of the diffusion ($\delta^2 = 0.05$), and that there is only one jump per year ($\lambda = 1$). This implies that the variance of the diffusion process is practically half of the total variance of the process. With these values, the Merton option price is $F = 5.9713$. As a reference, the Black-Scholes price *if* there were no jumps would be $F_e(\sigma^2)$=5.3396. However, this will not be the appraised option value of an investor who incorrectly believes that the stock price follows a diffusion process and estimates the volatility from historical data. As stated before, her estimation of the variance will be ν^2=0.10062 and her appraised option value $F_e(\nu^2)$=6.0711. In this case the investor will overprice the option by only 1.7%. However, the pricing error increases dramatically as there are larger and fewer jumps per year. For instance, when $\delta^2 = 5$ and $\lambda = 0.01$, Merton option price is $F = 5.4560$, while the investor appraised value will be $F_e(\nu^2)$=6.8168 (a 24.9% higher). Notice that in this case the variance of the diffusion part accounts only for 30.77% of the total variance of the process.

Table 1 also shows the appraised option values incorrectly computed by Ball and Torous (1985) using ν_M, $F_e(\nu_M^2)$. For the case $\delta^2 = 0.05$ and $\lambda = 1.00$, we have that $\nu_M^2 = 0.10$ and $F_e(\nu_M^2)$=6.06283, while $F_e(\nu^2)$=6.07110, just a 0.13% higher. However, as the volatility (and magnitude) of the jumps increases and their frequency decreases, the difference between $F_e(\nu_M^2)$ and $F_e(\nu^2)$ increases substantially. For example, when $\delta^2 = 5$ and $\lambda = 0.01$, $F_e(\nu_M^2)$=6.06283, but $F_e(\nu^2)$=6.81684. Hence, the investor using ν_M in the Black-Scholes formula would observe an overpricing of 11.12%, while the actual error (using ν) will be 24.9%.

An interesting question is whether the lack of significant differences between the Black-Scholes and Merton model prices found by Ball and Torous (1985) could be due to their miscalculation of the total volatility of the process. Using their parameter estimates, we find that the difference between ν and ν_M is insignificant in most of the stocks. Thus, using the correct volatility measure apparently will not change their results. However, rows 5-16 of Table 1 show that if we relax Ball and Torous' assumption that the mean relative jump size is zero, the difference in option prices can be large. For example, when investors expect one jump per year that will produce a mean relative jump in stock price of -20% (row 11), $F = 6.6872$, $F_e(\nu^2) = 6.0628$, and $F_e(\nu_M^2)$=0.8066. That is, an investor using incorrectly the Black-Scholes formula and miscalculating the volatility of the process will underprice the option by 9.3%, while that using the correct volatility she will overprice it by 1.8%. Hence, the restriction imposed on the jump size together with the miscalculation of the volatility could partially explain the results of Ball and Torous (1985).

We can further analyze the pricing errors standardizing the variables as in Merton (1976a). He also considers the special case in which $\mu = -\frac{\delta^2}{2}$. Substituting $\kappa = 0$ into equation (2), the option pricing formula simplifies to

$$F = \sum_{n=0}^{\infty} \frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!} W(S(t), \tau, E, \sigma_n^2, r), \tag{6}$$

where $\sigma_n^2 = \sigma^2 + \frac{n}{\tau}\delta^2$.

If we define $W'(S, \tau) \equiv W(S, \tau, 1, 1, 0)$, it is easy to show that

$$W'(X, \tau_n) = \frac{W(S, \tau, E, \sigma_n^2, r)}{Ee^{-r\tau}},$$

where $X \equiv \frac{S}{Ee^{-r\tau}}$ and $\tau_n \equiv \sigma_n^2 \tau$.

With this notation, we rewrite equation (6) as

$$F = Ee^{-r\tau} \sum_{n=0}^{\infty} \frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!} W'(X, \tau_n). \tag{7}$$

Similarly, we can express the investor appraisal value of the option using ν_M as

$$F_e(\nu_M^2) \equiv W(S, \tau, E, \nu_M^2, r) = Ee^{-r\tau} W'(X, \bar{\tau}), \tag{8}$$

where $\bar{\tau} \equiv \nu_M^2 \tau$, and can be considered as a maturity measure of the option.

Merton (1976a) defines two other standardized variables: Γ, a variance measure[2], and Λ, the expected number of jumps during the life of the option divided by the maturity measure $\bar{\tau}$. Their expressions are given by

$$\begin{aligned} \Gamma &\equiv \frac{\lambda\delta^2}{\nu_M^2}, \\ \Lambda &\equiv \lambda\frac{\tau}{\bar{\tau}}. \end{aligned}$$

From these definitions it is easy to show that $\tau_n = (1-\Gamma)\bar{\tau} + n\frac{\Gamma}{\Lambda}$ and $\lambda\tau = \Lambda\bar{\tau}$.

Finally, from equations (7) and (8) we can express the option prices normalized by the present value of the exercise price as

$$f = \sum_{n=0}^{\infty} \frac{e^{-\Lambda\bar{\tau}}(\Lambda\bar{\tau})^n}{n!} W'\left(X, (1-\Gamma)\bar{\tau} + n\frac{\Gamma}{\Lambda}\right), \text{and} \quad (9)$$

$$f_{eM} \equiv \frac{F_e(\nu_M^2)}{Ee^{-r\tau}} = W'(X, \bar{\tau}). \quad (10)$$

To analyze how the pricing error changes as there are fewer but larger jumps in the stock price, we study three cases. We use a maturity measure of ($\bar{\tau}$) of 0.05, representing, for example, a two-month option with annual ν_M^2 of 0.30.

Figure 2 shows the absolute and relative normalized pricing errors, defined as $f - f_e$ and $\frac{f-f_e}{f_e}$ respectively, for different stock prices. We consider relatively small and frequent jumps in stock prices: $\Gamma = 0.1, \Lambda = 5$. For our option, this implies that there are 1.5 jumps per year with annual variance (δ^2) of 0.02 and a volatility of the diffusion part of 51.96%. We plot the error for both ν_M and ν. We see that the errors computed with ν_M are very similar to those obtained with ν. If the investor incorrectly prices options with the Black-Scholes formula when the stock price follows a jump-diffusion process, she will

[2]Notice that Γ does not represent the variance of the jump process divided by the total variance of the jump-diffusion process, as stated by Merton (1976a).

underprice DOTM and DITM options and overprice NTM options. The relative error is very small for DITM and NTM options (less than 0.5%), but it increases dramatically for DOTM options.

Insert Figure 2 about here.

In Figure 3 we also consider small jumps, but they occur less frequently and with higher variance: $\Gamma = 0.1, \Lambda = 0.01$. In our example, this implies $\lambda = 0.003, \delta^2 = 10$, and $\sigma = 51.96\%$. We see that now the errors computed with ν_M are much smaller than the real ones (those obtained with ν). For example, when the standardized stock price, X, is 0.61 the relative pricing error with ν_M is -1.27%, while the error with ν is -53.54%.

Insert Figure 3 about here.

In Figure 4 we have very large but infrequent jumps: $\Gamma = 0.9, \Lambda = 0.01$, which implies $\lambda = 0.003, \delta^2 = 90$, and $\sigma = 17.32\%$. We observe that in this case the pricing errors between the Merton (1976b) and the Black-Scholes models are much larger, and that the difference between the errors computed with ν_M and ν increases substantially.

Insert Figure 4 about here.

From these figures, it seems that the impact of using the correct volatility when comparing Black-Scholes and Merton model prices is significant only for unreasonable parameter values. However, this is not necessarily the case. In Figure 5 we study the scenario in which $\bar{\tau} = 0.0492$, $\Gamma = 0.3670$, and $\Lambda = 1.8084$. This represents, for example, a one-year option with an annual Merton's volatility rate (ν_M) of 0.2218, when the number of jumps per year (λ) is 0.0890 and the volatility of the diffusion part (σ) is 0.1765. These values are obtained from Andersen and Andreasen (1999), who estimate the jump-diffusion parameters for a sample of S&P 500 index options, without assuming that $\mu = -\frac{\delta^2}{2}$. Notice than in this case, the total volatility of the process (ν) is 0.3459, considerably higher than Merton's volatility rate. An investor using ν_M in the Black-Scholes formula will observe a pricing error that is larger (in absolute value) for DITM options and smaller for most of the other options than what she would actually observe in the market (using ν). Moreover, if the normalized stock price (X) is greater than 0.94, the Black-Scholes model with ν_M underprices the option, while with ν the model overprices the option. For example, when $S = 100$ and $E = 100$ ($X = 1.10$) an investor using ν_M in the Black-Scholes formula will underprice the option by 7.85%, when in fact the model overprices it by 18.33%. Thus the practical relevance of the miscalculation of the total variance of the jump-diffusion process may not be insignificant.

Insert Figure 5 about here.

Finally, in Tables 2–5 we replicate Tables I–IV of Merton (1976a) using ν instead of ν_M. For different standardized parameter values, these tables report the stock prices at which Black-Scholes price equals Merton price, the stock prices at which there is a local maximum of the absolute value of the pricing error, the maximum percentage overestimate of option price using Black-Scholes model, and the percentage underestimate of option price at stock price equal to 0.5 of present value of exercise price, respectively.

In Table 2 we see that the crossover points differ from those reported by Merton. For example, when $\bar{\tau} = 0.30$, $\Gamma = 1$, and $\Lambda = 5$, Black-Scholes and Merton option prices are equal when the normalized stock price is 0.572, while Merton (1976a) reports a value of 0.634 (more than 10% higher). Since DOTM options, significantly underpriced by the Black-Scholes formula, turns sharply into significantly overpriced options, a small error in the computation of the crossover points can have an important effect on the study of these options.

In the tables, we also observe that the percentage pricing error is very large when there are few but large jumps (Λ small and Γ large) for short maturity options ($\bar{\tau}$ small). These are also the cases in which the pricing errors reported here differ the most from those of Merton (1976a).

Insert Tables 2–5 about here.

3 Conclusions

If the true process describing the dynamics of stock returns is a jump-diffusion process, but an investor incorrectly believes that stock returns follow a diffusion process, she will use the Black-Scholes formula to price options. If she estimates the volatility of the diffusion process from time series data, she will estimate the total volatility of the jump-diffusion process. When she uses this volatility in the Black-Scholes formula, she will observe a pricing error. Merton (1976a), Ball and Torous (1985), Jorion (1988) and Amin (1993) have analyzed this error. However, instead of using the total volatility of the jump-diffusion process, they use a smaller volatility rate.

In this paper we analyze the price difference between Black-Scholes and Merton models using the total volatility of the jump-diffusion process in the Black-Scholes formula. We show that the stock prices at which the pricing "error" is zero are different from those reported by Merton (1976a). Consequently, some options underpriced with the Black-Scholes formula in Merton's paper are in fact overpriced and vice versa. We also show that the pricing error can be smaller and larger than what was previously reported. Although the difference in pricing errors is very large for unreasonable parameter estimates, we find that it is non negligible for reasonable ones.

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Table 1: **Call option prices for different parameters of the jump-diffusion process**.

	δ^2	λ	μ	ν_M^2	ν^2	F	$F_e(\sigma^2)$	$F_e(\nu_M^2)$	$F_e(\nu^2)$
	0.05	1.00	-0.025	0.10	0.10062	5.9713	5.3396	6.0628	6.0711
$k = 0$	0.50	0.10	-0.250	0.10	0.10625	5.6979	5.3396	6.0628	6.1447
	5.00	0.01	-2.500	0.10	0.16250	5.4560	5.3396	6.0628	6.8168
	0.05	1.00	0.070	0.10	0.10494	5.9647	5.3396	6.0628	6.1277
$k = 0.1$	0.50	0.10	-0.155	0.10	0.10239	5.6826	5.3396	6.0628	6.0944
	5.00	0.01	-2.405	0.10	0.15783	5.4573	5.3396	6.0628	6.7648
	0.05	1.00	0.157	0.10	0.12475	6.1554	5.3396	6.0628	6.3778
$k = 0.2$	0.50	0.10	-0.068	0.10	0.10046	5.6758	5.3396	6.0628	6.0689
	5.00	0.01	-2.318	0.10	0.15372	5.4587	5.3396	6.0628	6.7186
	0.05	1.00	-0.130	0.10	0.11699	6.2055	5.3396	6.0628	6.2817
$k = -0.1$	0.50	0.10	-0.355	0.10	0.11263	5.7234	5.3396	6.0628	6.2266
	5.00	0.01	-2.605	0.10	0.16788	5.4550	5.3396	6.0628	6.8759
	0.05	1.00	-0.248	0.10	0.16158	6.6872	5.3396	6.0628	6.8066
$k = -0.2$	0.50	0.10	-0.473	0.10	0.12239	5.7603	5.3396	6.0628	6.3488
	5.00	0.01	-2.723	0.10	0.17416	5.4541	5.3396	6.0628	6.9440

This table replicates part of Table IV of Ball and Torous (1985). They assume that the mean relative jump size of the stock (k) is zero. The other parameters of the model are $S(0) = 38, \tau = 0.5, E = 35, \sigma^2 = 0.05$, and $r = 0.10$, where σ is the volatility of the diffusion process. The total volatility per unit of time of the jump-diffusion process is denoted by ν, whereas ν_M is the volatility used by Ball and Torous. F represents the Merton model price, and F_e is the investor appraised option value using the Black-Scholes formula.

Table 2: **Normalized stock price at which Black-Scholes and Merton option prices are equal**.

	$\bar{\tau}$	Γ 0.10	0.25	0.40	0.50	0.75	1.00
$\Lambda = 5$	0.05	1.274	1.284	1.286	1.284	1.286	1.235
		0.785	0.779	0.778	0.779	0.789	0.810
	0.10	1.411	1.419	1.417	1.411	1.381	1.324
		0.709	0.705	0.706	0.709	0.724	0.756
	0.20	1.661	1.667	1.661	1.651	1.601	1.511
		0.602	0.600	0.602	0.606	0.625	0.662
	0.30	1.914	1.920	1.913	1.901	1.842	1.749
		0.523	0.521	0.523	0.526	0.543	0.572
$\Lambda = 10$	0.05	1.267	1.271	1.269	1.265	1.246	1.207
		0.790	0.787	0.788	0.790	0.803	0.829
	0.10	1.405	1.407	1.403	1.397	1.366	1.306
		0.712	0.711	0.713	0.716	0.732	0.766
	0.20	1.655	1.657	1.652	1.644	1.607	1.546
		0.604	0.603	0.605	0.608	0.622	0.647
	0.30	1.907	1.910	1.905	1.897	1.860	1.804
		0.524	0.524	0.525	0.527	0.538	0.554
$\Lambda = 20$	0.05	1.263	1.264	1.261	1.257	1.238	1.198
		0.792	0.791	0.793	0.795	0.808	0.835
	0.10	1.402	1.403	1.399	1.394	1.372	1.330
		0.713	0.713	0.715	0.717	0.729	0.752
	0.20	1.652	1.653	1.649	1.645	1.624	1.589
		0.605	0.605	0.606	0.608	0.616	0.629
	0.30	1.904	1.905	1.902	1.898	1.877	1.844
		0.525	0.525	0.526	0.527	0.533	0.542

This table replicates Table I of Merton (1976a), using the total volatility of the jump-diffusion process. Λ, Γ, and $\bar{\tau}$ represent measures of the number of jumps per year, the variance of the process, and the maturity of the option, respectively.

Table 3: **Normalized stock prices at which there is a local maximum of the absolute value of the dollar error in option price**.

	$\Lambda = 5$					$\Lambda = 10$					$\Lambda = 20$				
	Γ					Γ					Γ				
$\bar{\tau}$	0.10	0.25	0.50	0.75	1.00	0.10	0.25	0.50	0.75	1.00	0.10	0.25	0.50	0.75	1.00
0.05	1.520	1.539	1.540	1.520	1.501	1.506	1.514	1.503	1.471	1.438	1.499	1.501	1.488	1.456	1.420
	1.010	1.010	1.009	1.008	1.000	1.009	1.009	1.009	1.007	1.000	1.009	1.009	1.008	1.007	1.000
	0.670	0.662	0.663	0.674	0.685	0.675	0.673	0.678	0.693	0.711	0.678	0.678	0.684	0.699	0.719
0.10	1.805	1.819	1.803	1.754	1.707	1.792	1.797	1.775	1.724	1.672	1.785	1.787	1.771	1.736	1.704
	1.019	1.019	1.018	1.014	1.000	1.019	1.019	1.017	1.014	1.000	1.018	1.018	1.017	1.014	1.000
	0.573	0.569	0.575	0.592	0.613	0.577	0.576	0.583	0.601	0.624	0.579	0.578	0.584	0.596	0.610
0.15	2.073	2.086	2.058	1.987	1.920	2.060	2.065	2.040	1.984	1.937	2.054	2.055	2.039	2.003	1.965
	1.029	1.029	1.027	1.021	1.000	1.029	1.029	1.027	1.022	1.000	1.028	1.028	1.027	1.024	1.000
	0.507	0.504	0.511	0.531	0.555	0.509	0.509	0.515	0.531	0.548	0.511	0.510	0.515	0.525	0.536
0.20	2.341	2.353	2.319	2.236	2.166	2.328	2.332	2.306	2.249	2.203	2.321	2.323	2.307	2.270	2.227
	1.040	1.040	1.037	1.029	1.000	1.040	1.039	1.037	1.031	1.000	1.039	1.039	1.038	1.034	1.000
	0.455	0.453	0.461	0.480	0.502	0.457	0.457	0.462	0.475	0.489	0.458	0.458	0.462	0.470	0.480
0.25	2.615	2.628	2.590	2.503	2.442	2.601	2.606	2.580	2.521	2.470	2.594	2.597	2.580	2.543	2.497
	1.052	1.052	1.048	1.038	1.000	1.051	1.051	1.048	1.042	1.000	1.051	1.051	1.049	1.046	1.003
	0.413	0.412	0.418	0.435	0.453	0.415	0.414	0.419	0.430	0.442	0.416	0.416	0.419	0.425	0.434
0.30	2.900	2.913	2.874	2.785	2.735	2.885	2.891	2.864	2.804	2.746	2.878	2.880	2.864	2.826	2.778
	1.064	1.064	1.060	1.048	1.000	1.063	1.063	1.060	1.053	1.000	1.063	1.063	1.061	1.058	1.035
	0.378	0.376	0.382	0.397	0.411	0.379	0.379	0.383	0.392	0.403	0.380	0.380	0.382	0.388	0.395

This table replicates Table II of Merton (1976a), using the total volatility of the jump-diffusion process.

Table 4: **Maximum percentage overestimate of option price using Black-Scholes model: Normalized stock price and percentage error** $[(f - f_e)/f_e]$.

	$\bar{\tau}$	Γ 0.10	0.25	0.40	0.50	0.75	1.00
$\Lambda = 5$	0.05	0.894	0.889	0.890	0.893	0.912	1.000
		-0.646	-3.613	-8.538	-12.786	-26.773	-54.887
	0.10	0.853	0.850	0.854	0.859	0.885	1.000
		-0.367	-2.168	-5.335	-8.177	-18.069	-40.256
	0.20	0.788	0.787	0.792	0.800	0.836	1.000
		-0.215	-1.308	-3.280	-5.073	-11.350	-24.991
	0.30	0.732	0.732	0.738	0.746	0.786	1.000
		-0.163	-0.999	-2.517	-3.895	-8.623	-17.241
$\Lambda = 10$	0.05	0.898	0.897	0.899	0.903	0.921	1.000
		-0.341	-2.008	-4.944	-7.597	-17.016	-39.325
	0.10	0.856	0.856	0.860	0.865	0.890	1.000
		-0.188	-1.144	-2.877	-4.470	-10.212	-23.866
	0.20	0.790	0.790	0.795	0.800	0.828	1.000
		-0.109	-0.670	-1.699	-2.643	-5.945	-11.597
	0.30	0.734	0.734	0.738	0.743	0.768	0.854
		-0.082	-0.507	-1.287	-2.002	-4.459	-7.647
$\Lambda = 20$	0.05	0.900	0.900	0.903	0.907	0.924	1.000
		-0.175	-1.064	-2.680	-4.173	-9.641	-23.281
	0.10	0.858	0.858	0.861	0.865	0.884	1.000
		-0.095	-0.588	-1.494	-2.332	-5.322	-10.949
	0.20	0.791	0.792	0.794	0.797	0.812	0.854
		-0.055	-0.340	-0.864	-1.348	-3.032	-5.319
	0.30	0.735	0.736	0.738	0.740	0.752	0.778
		-0.041	-0.255	-0.651	-1.015	-2.273	-4.002

This table replicates Table III of Merton (1976a), using the total volatility of the jump-diffusion process.

Table 5: **Percentage underestimate $[(f - f_e)/f_e]$ of option price using Black-Scholes model at stock price equal to 0.5 of present value of exercise price.**

		Γ					
	$\bar{\tau}$	0.10	0.25	0.40	0.50	0.75	1.00
$\Lambda = 5$	0.05	63.463	426.738	1056.523	1569.914	2957.016	4207.807
	0.10	6.360	36.482	84.757	123.425	226.968	320.767
	0.15	1.721	9.909	23.688	35.467	70.389	103.661
	0.20	0.611	3.546	8.719	13.437	29.374	46.401
	0.25	0.223	1.291	3.266	5.207	12.905	22.651
	0.30	0.058	0.321	0.863	1.501	4.915	10.535
$\Lambda = 10$	0.05	29.205	195.346	499.251	762.643	1558.273	2437.933
	0.10	3.247	19.364	46.681	69.571	135.197	202.073
	0.15	0.889	5.323	13.132	20.028	41.613	64.871
	0.20	0.317	1.909	4.804	7.478	16.730	27.941
	0.25	0.117	0.703	1.802	2.866	6.980	12.816
	0.30	0.031	0.187	0.498	0.833	2.431	5.337
$\Lambda = 20$	0.05	13.866	90.940	235.189	364.929	785.266	1305.925
	0.10	1.640	9.996	24.706	37.495	76.805	121.910
	0.15	0.452	2.762	6.934	10.697	23.103	38.400
	0.20	0.161	0.990	2.516	3.929	8.903	15.704
	0.25	0.060	0.366	0.940	1.486	3.536	6.696
	0.30	0.016	0.100	0.262	0.426	1.129	2.453

This table replicates Table IV of Merton (1976a), using the total volatility of the jump-diffusion process.

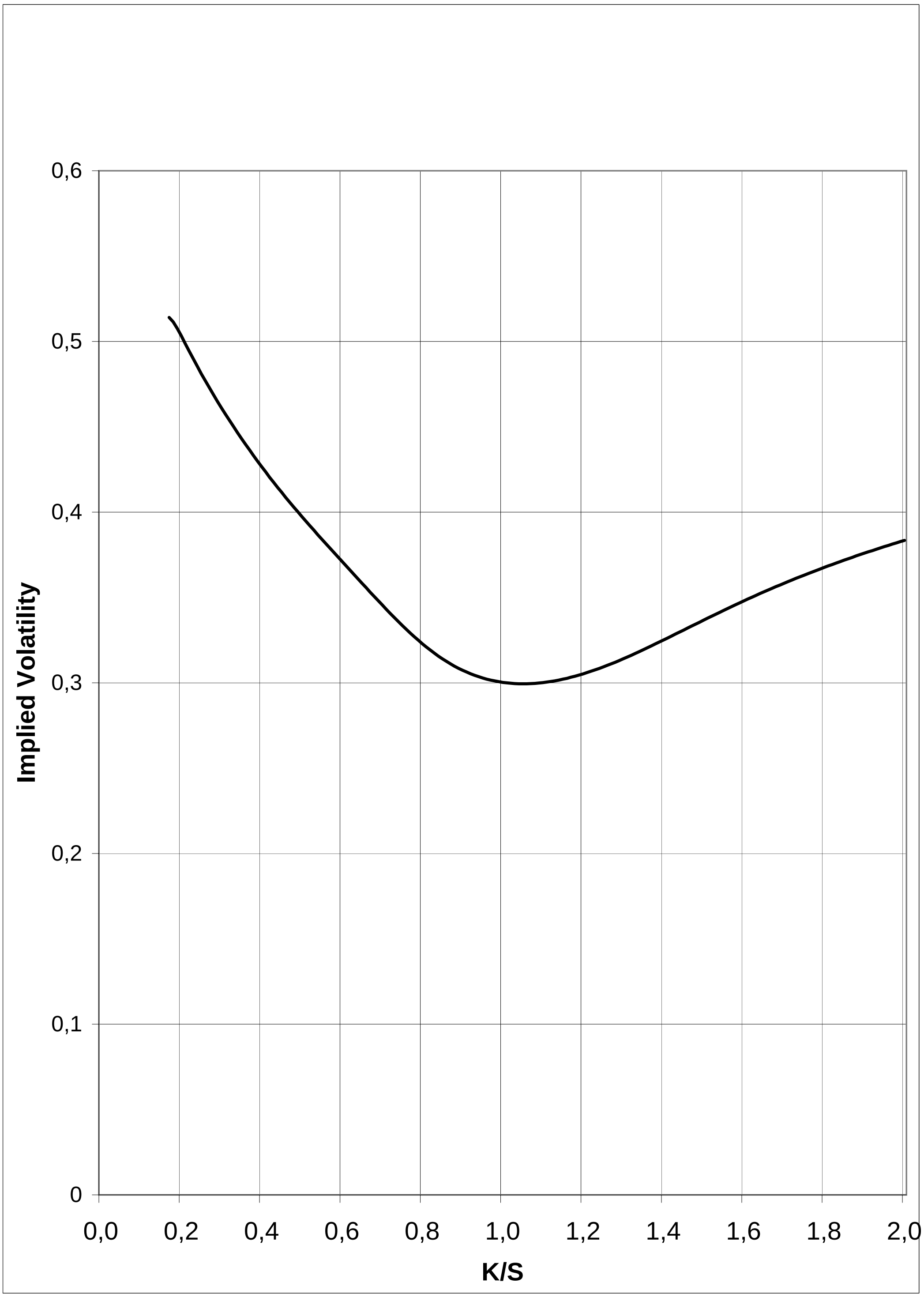


Figure 1: **Implied Black-Scholes Volatility from Merton (1976b) call option prices**. The parameters of the model are $\tau = 0.5, \sigma^2 = 0.05, r = 0.10, \mu = -0.025, \delta^2 = 0.05$, and $\lambda = 1$.

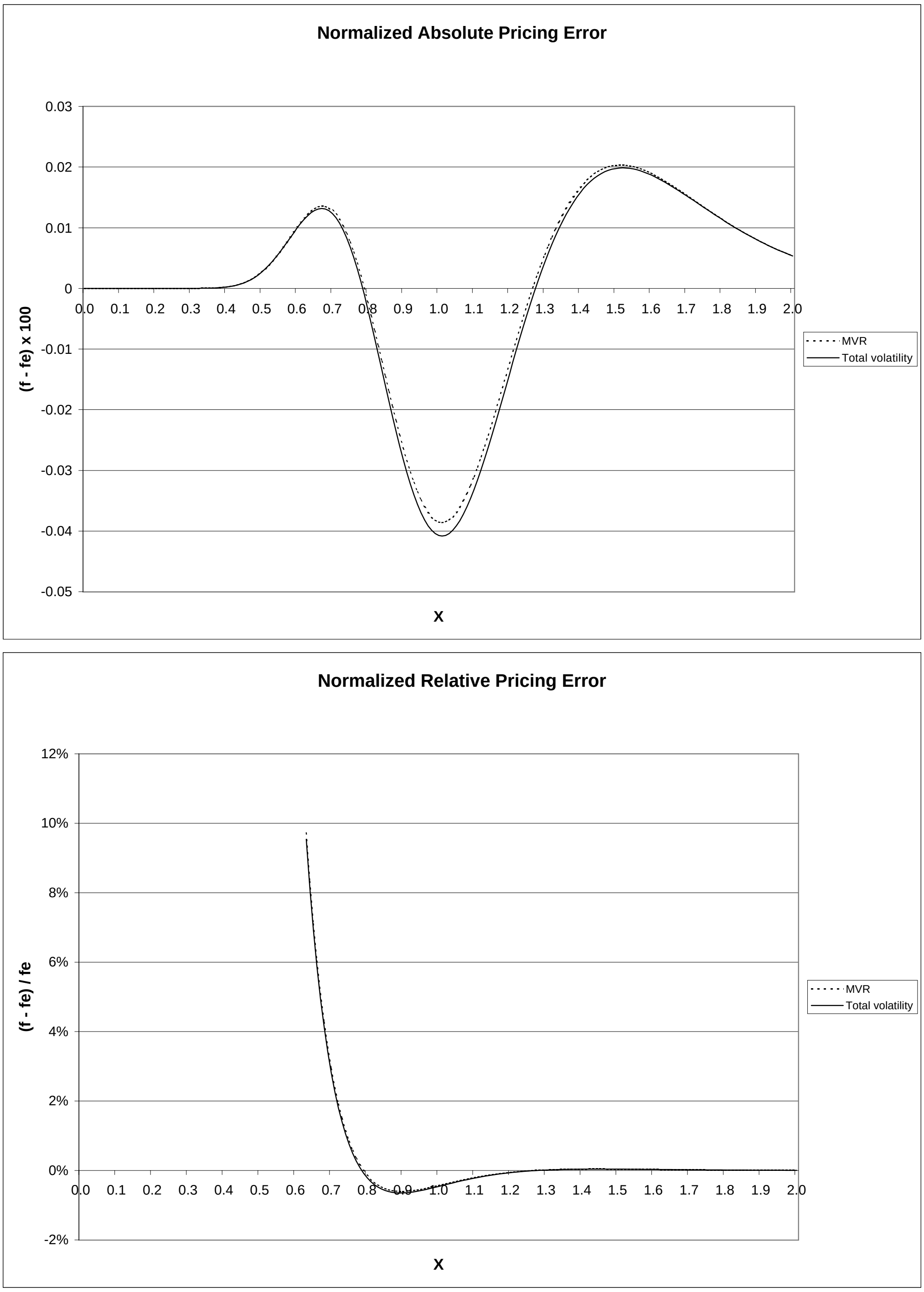
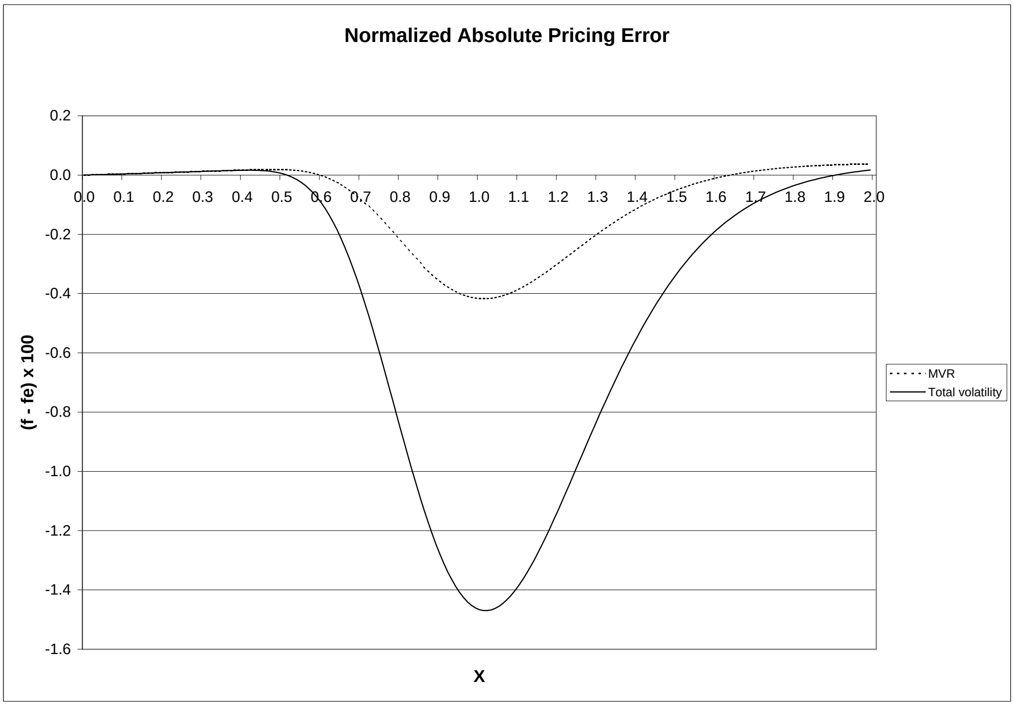


Figure 2: **Option price error when jumps are small and frequent**. MVR stands for Merton's volatility rate. Stock and option prices are expressed in units of the present value of the exercise price. X is the normalized stock price, f is the normalized Merton (1976b) call option price, and f_e is the normalized investor appraisal of the option value when using the Black-Scholes formula. The parameters of the model are $\bar{\tau} = 0.05, \Gamma = 0.1$ and $\Lambda = 5$.



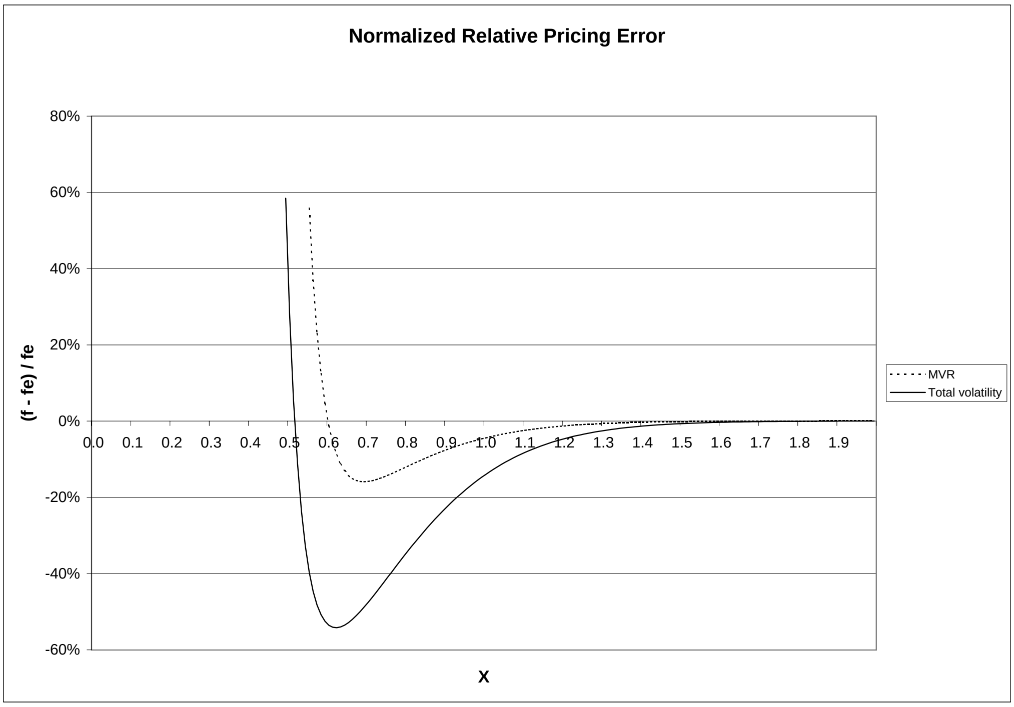


Figure 3: **Option price error when jumps are large and infrequent**. MVR stands for Merton's volatility rate. Stock and option prices are expressed in units of the present value of the exercise price. X is the normalized stock price, f is the normalized Merton (1976b) call option price, and f_e is the normalized investor appraisal of the option value when using the Black-Scholes formula. The parameters of the model are $\bar{\tau} = 0.05, \Gamma = 0.1$ and $\Lambda = 0.01$.

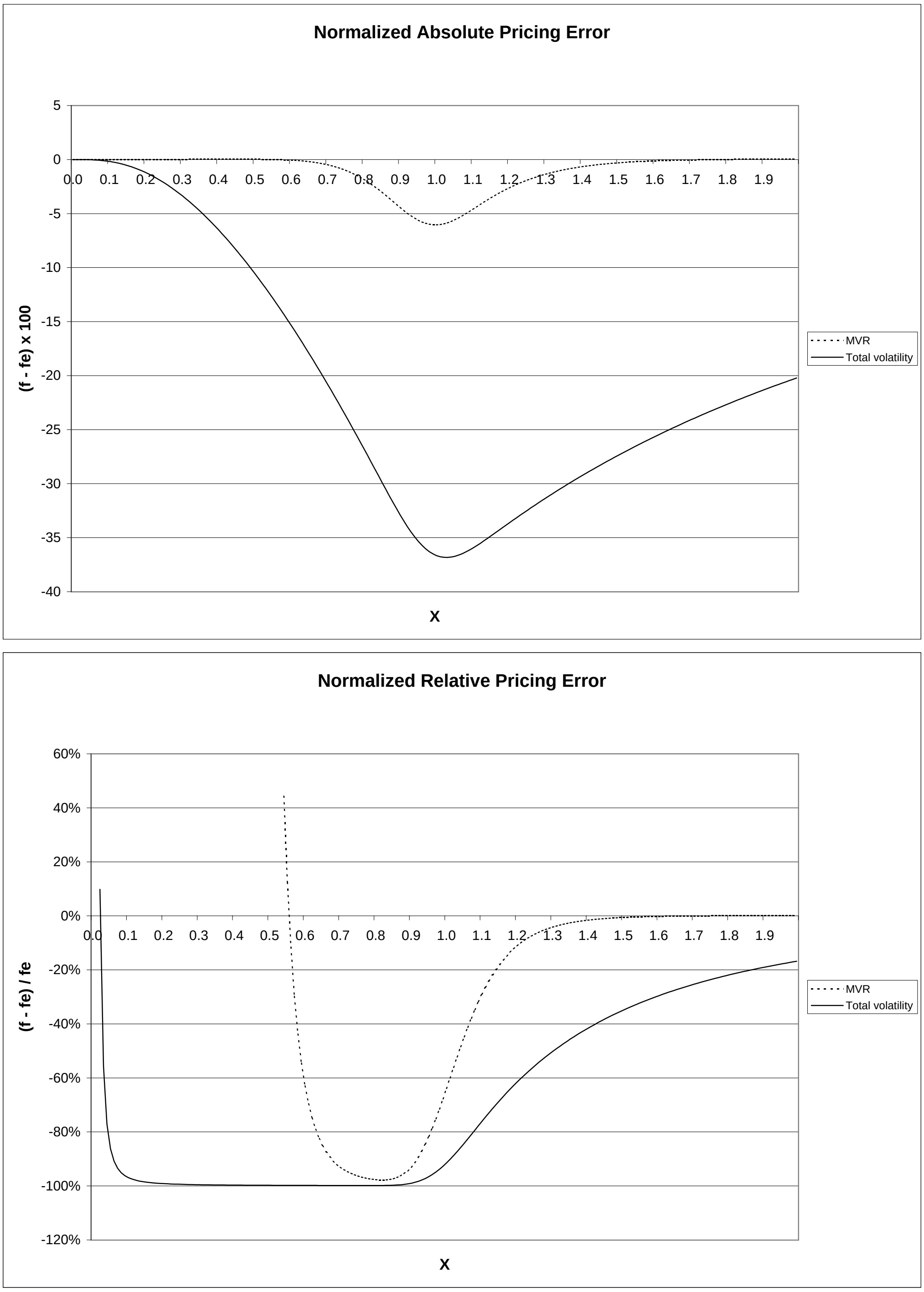
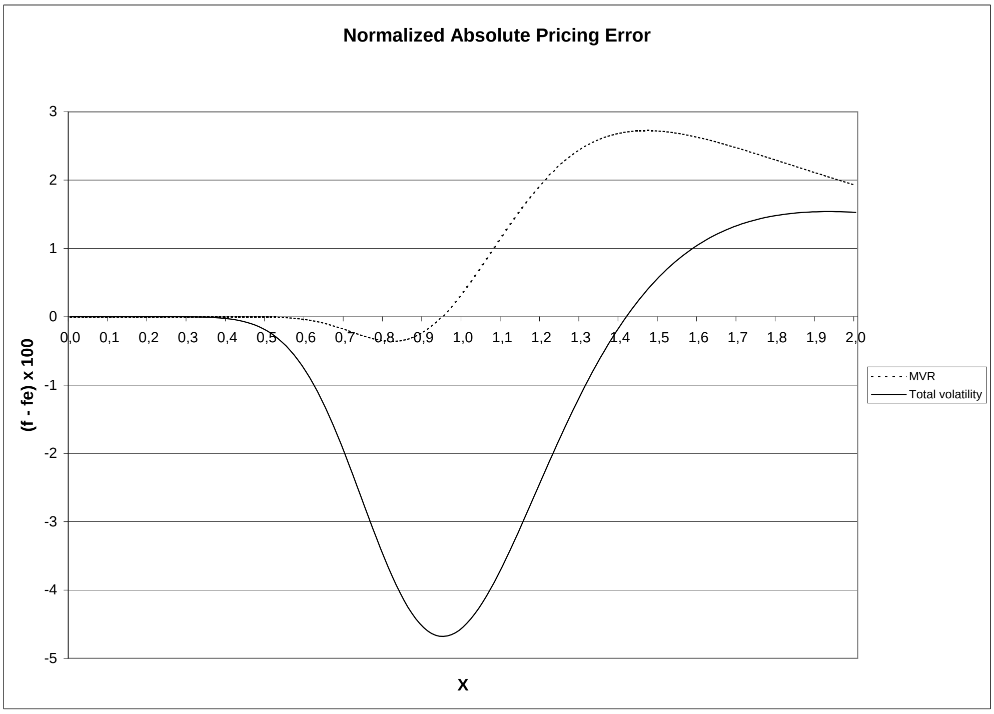


Figure 4: **Option price error when jumps are very large and infrequent**. MVR stands for Merton's volatility rate. Stock and option prices are expressed in units of the present value of the exercise price. X is the normalized stock price, f is the normalized Merton (1976b) call option price, and f_e is the normalized investor appraisal of the option value when using the Black-Scholes formula. The parameters of the model are $\bar{\tau} = 0.05, \Gamma = 0.9$ and $\Lambda = 0.01$.



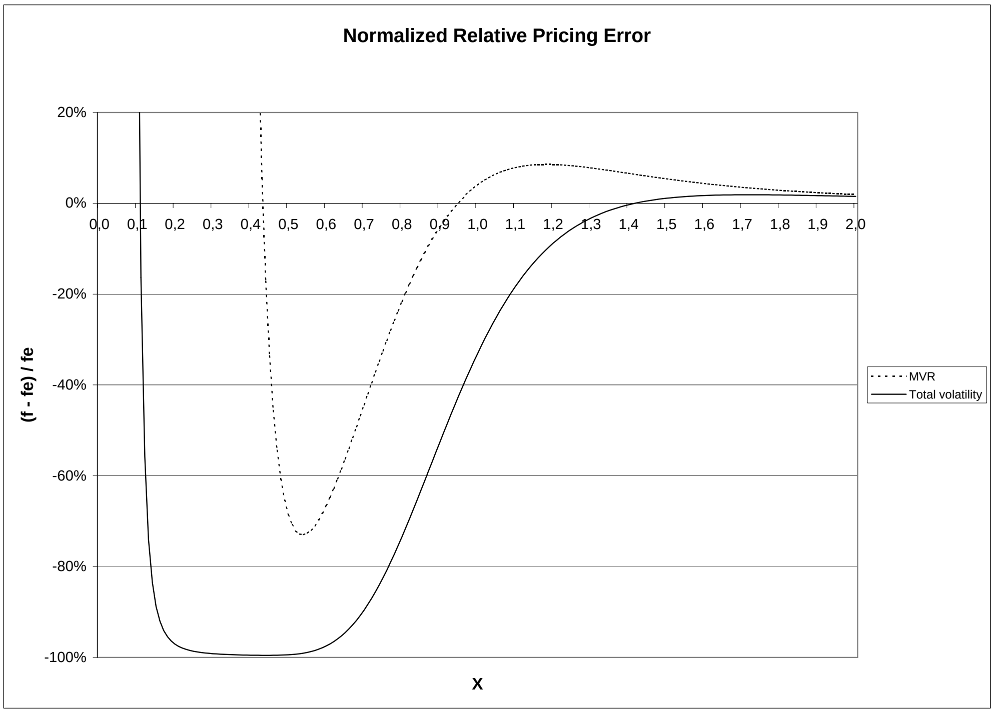


Figure 5: **Option price error for actual parameter values**. MVR stands for Merton's volatility rate. Stock and option prices are expressed in units of the present value of the exercise price. X is the normalized stock price, f is the normalized Merton (1976b) call option price, and f_e is the normalized investor appraisal of the option value when using the Black-Scholes formula. The parameters of the model are $\bar{\tau} = 0.0492, \Gamma = 0.367$ and $\Lambda = 1.808$, and are obtained from Andersen and Andreasen (1999).