AN ANALYSIS OF THE ROBUSTNESS OF GENETIC ALGORITHM (GA) METHODOLOGY IN THE DESIGN OF TRADING SYSTEMS FOR THE STOCK EXCHANGE*

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Laura Núñez

Instituto de Empresa
Research Management
C/ Pinar, 7
28006, Madrid – Spain
laura.nunez@ie.edu

Abstract

This paper analyzes the robustness of Genetic Algorithms (GAs) technique for its application in the field of trading systems design for the Stock Exchange. The functioning of the GA is driven by the control parameters: crossover and mutation probabilities, number of generations, and size of population. Whether the results generated by the application of GAs to a specific problem are conditioned by the value assess to these parameters, becomes a main research field. The purpose of this paper is to develop a sensibility analyses about the dependency of the GA to the value of these parameters. The sensibility analyses is developed in part by a hierarchic GA (a GA which is used to the optimisation of the control parameters of a second GA which is used to design the trading system). The results find that the GAs are a very robustness technique when logical ranges are considered for these parameters (taken into account that there is a high level of complementation between them), with a wide optimisation capacity.

Keywords

Genetic algorithms, control parameters, trading systems.

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Introduction

This paper aims to analyse the robustness of the GA technique for its application in the field of trading systems design for the Stock Exchange. At the same time, the optimisation power of the GA methodology will be tested.

GA are a search technique, of general character and domain, inspired by evolutionary mechanisms and theories, natural selection and genetics and presenting characteristics which, for specific problems, make this technique superior to the traditional heuristic methods based on calculus or random or enumerative procedures. These characteristics incorporate the possibility of carrying out searches in large spaces, mainly due to their explicit and implicit parallelism. It hardly requires any knowledge of the problem to be solved and, above all, and contrarily to econometric models, it does not imply any type of restrictive hypothesis on the data or functions characterising the problem (for instance, single peaked distributions, or continuous, derivable, or linear functions, etc.). When GA are properly designed, they are better suited than other techniques in overcoming the problem of local optimums.

The technical framework that defines the analytical basis of GA functioning was initially developed in the mid seventies by John Holland (1975), in relation to the application of binary GA. This framework refers to the "schemata theorem", "implicit parallelism" and "deception problem", whose analysis is beyond the objectives of this paper. Goldberg (1989a) presents an excellent review of the mathematical foundations that support GA functioning. Other references to mathematical foundations of GA are: Davis (1991); Michalewicz (1996); Whitley (1992); Stephens et al (1999); Grefenstette and Baker (1989); Grefenstette (1991)

There are two essential actions in the GA procedure: the genetic representation of solutions to the problem by vectors or chromosomes (often through the binary codification[1]) and the definition of the fitness function, the function which classifies the solutions in an orderly way on the basis of a known objective function.

The functioning of a traditional GA is structured into a series of steps: (1) the random generation of a group of potential solutions to a specific problem (the one to be solved); (2) the representation of these solutions by vectors or chromosomes through their binary codification; (3) the calculation of their corresponding fitness functions; (4) application of a series of mechanisms or operators which manipulate the information of the chromosomal chains or vectors in the appropriate direction in order to increase the value of their fitness functions (three mechanisms are normally used: a reproduction mechanism where solutions with a higher fitness function are more largely reproduced, a crossover mechanism where the information of the previously reproduced chains is exchanged, and a mutation mechanism where one of the values of the chromosomal chains is randomly substituted by another); (5) calculation of the fitness functions of the new set of chromosomes thus obtained; (6) application once again of the reproduction, crossover and mutation mechanisms in order to obtain a new set of chromosomes or solutions; (7) iterative repetition of steps 5 and 6 until the fitness functions of the set of solutions generated over a specific number of iterations can not be further improved or during a fixed number of generations (iterations).

It is necessary to take into account that the reproduction operator generates new spaces of solutions which only contain values or solutions identical to the initial ones, although in a different proportion since they have been weighted by their relative fitness function in the reproduction process. For this reason, the existence of some mechanism able to include new solutions is extremely important, since the global search space would otherwise be restricted to the random selection carried out to generate the first population.

The mechanisms which enable us to widen the diversity of the population, through the introduction of individuals different from those selected by random procedure at initial time, proceed from crossover and mutation operators. While the former allows the exchange of partial information among different individuals, the latter alters the information contained in an individual by varying some component of the codification of this solution, thus leading to a different individual or solution. The crossover operator is controlled by a parameter – the crossover probability – that indicates which percentage of individuals must be submitted to the exchange mechanism of genetic information. This parameter or crossover probability must take a value allowing the survival of part of the chromosomes selected in the reproduction process. The mutation operator is applied to a very reduced set of individuals and is controlled by the mutation probability. Its aim is to randomly select a series of individuals (depending on this probability) in order to substitute one of his genetic components by an alternative one, which in the case of binary GA leads to substitute "zero" by "one" or vice versa.

The correct design of the GA is not a trivial task. The functioning of the GA is driven by the control parameters: crossover and mutation probabilities, number of generations, and size of population. Whether the results generated by the application of GA to a specific problem are conditioned by the value assessed to these parameters and the choice of operators becomes a main research field.

Various empirical analyses have detected that the appropriate value for the parameters seems to depend on the typology of problems to be solved. This implies that, when carrying out a specific GA application, it is essential to incorporate, or to previously perform, some kind of process in order to set them. The technique normally used is a "manual" approach through repeated executions of GA which introduce different values in all the parameters. Sometimes, more rigorous techniques are used, such as the GA application to the optimisation of these parameters, or techniques which explore the space of solutions in an enumerative way. However, the problem lies in the great amount of time and resources which these more rigorous approaches require. GA methodology is computational intensive therefore, when it is applied to real complex problems, the time needed to get a solution is a key factor. *"It could take longer to derive parameter values tailored to one's problem than the time available for solving the problem itself"*. Davis (1989)

Another way of defining the parameters, which has emerged from relatively recent research, is the one enabling the parameters to adapt themselves while the GA execution is taking place, Davis (1989) and Vallé and Basar (1999). In this way, although initial values are not very adequate, this problem is solved by allowing them to evolve. The disadvantage in this case is the need to predefine another type of parameter, related to the adaptation mechanism itself which will direct the self-estimation of the original parameters – crossover and mutation probabilities.

For an introductory reference in regard to the contributions of the literature to the selection of operators and control parameters see Haupt, R.L. & Haupt, S.E. (1998). Detailed analyses are presented in Syswerda (1989); Schaffer et al. (1989); Eshelman, Caruana and Schaffer (1989); Davis (1989; Fogarty (1989); Goldberg (1989b); Whitley (1989); Goldberg and Deb (1991); Spears and De Jong (1991); Liepins and Vose (1992); Vose (1994); Ronald (1997); Agrawal (1999); Agrawal and Deb (1999); Vallé and Basar (1999); Michalewicz (1996); Vose and Whitley (1998); and Fogel (2000).

The purpose of this paper is to develop a sensibility analysis about the dependency of the GA to the value of these parameters when it is used in the design of a simple trading system (focused on the crossing of two moving averages) for the Spanish Stock Exchange. Some references in regard to the use of GA in the design of trading systems are Bauer (1994); Karjalainen (1994); or Allen and Karjalainen (1999).

The structure of this paper is organised in the following way. Section 2 describes data and the GA built. Section 3 introduces the statistic approach and methodology used to test the results. Section 4 analyses the sensibility of the GA to the control parameters (number of individuals, number of generations, reproduction operator, crossover and mutation probabilities). The optimisation power of the GA is also tested. Section 5 presents the results obtained by the trading systems designed genetically when they are applied to data out of the sample of stocks and indexes quoted in the Madrid Stock Exchange[2]. Finally section 6 summarizes the main findings of the research and provides topics for further research.

Data and GA Used for Sensibility Analysis

The sensibility analysis to the control parameters is carried out for a GA designed to solve the problem of the optimisation of a simple trading system based on the technical indicator "Crossing of two Moving Averages" (CMA) which is used to generate buy and sell signals. The trading systems will be applied over the closing prices of a maximum of 25 stocks and indexes quoted in the Madrid Stock Exchange for the period between 29-05-1990 and 15-05-1995 (1,232 trading days).

The GA is programmed to find the number of sessions that have to be used in each moving average with the aim of maximizing the return obtained by the CMA system. The ranges allowed are 1-64 and 1-256 sessions for the shorter and the longer moving averages respectively[3]. These ranges lead to 16,384 different rules. It is a small number compared to the combinatorial explosion produced in more complex trading systems formed by multiple indicators. However, and for this reason, it allows the enumerative optimisation of the CMA rules, which will be used to test the power of the GA methodology for optimisation processes.

The GA is written in the programming language C[4] with the following features:

- Concatenated chromosomes with binary codification, which represent two variables related to the number of sessions to be used in each one of the two moving averages. The number of genes of these variables depends on the size allowed for the length of the moving averages. For a length of 64, six genes are needed ($2^6 = 64$), and 8 genes for a length of 256 ($2^8 = 256$).

- Reproduction is applied by the roulette algorithm. A roulette is designed in which the various options are represented by the different codified values of the individuals who integrate the partial search space or population, and the weight percentage of each solution in the roulette is the value of its relative fitness function in regard to the population fitness function. This roulette is used to randomly select codified values or chromosomes which engender successors for the next generation, the new individuals being constituted through an exact copy of the progenitors selected in the roulette. If the number of individuals which make up the successive populations is sufficiently big, their reproduction will tend to be proportional to their weight in the population.
- Binary crossover and mutation operators with probabilities of 60% of the individuals and 0.5% of the genes respectively.
- The number of individuals in each generation is set to 50, equivalent to 0.3% of the total population (search space: 16,384) and the number of generations is set to 50.
- The fitness function is the accumulated return obtained from the signals generated by the trading systems in the period of the sample.

The sensibility analysis will be focused on experiments based on the introduction of changes in the aforementioned parameters in the GA built for the design of CMA trading systems. The CMA trading systems will present the following features:

- They will take short[5] and long positions, but not out of the market positions.
- Transaction costs are not included.

The best trading system in each execution of the GA will be chosen by the return criterion. The risk factor is not taken into account. This is because the experiments do not aim to prove the utility of the trading systems analysed, but the robustness of the GA to the value of the control parameters.

Statistic Methodology

Three complementary tests are carried out for the sensibility analysis.

a.) Analysis of accumulated returns

The accumulated returns obtained by the selected CMA trading systems designed by the different versions of the GA[6] are compared among themselves in order to study the robustness of the GA. For the application of this test only five stocks are chosen: Ibex-35 index; Endesa; Repsol; Telefónica; and Iberdrola. This is because a statistic test needs at least a length of data of about 35[7], therefore each GA version has to be executed 35 times for each one of the stocks, with the associated amount of time that these executions require.

b.) Analysis of daily returns

A second test with daily returns (as a proxy to the accumulated returns) is carried out. In this case only one execution of each GA version is needed for each one of the stocks, therefore the test can be applied to a greater number of stocks with less computational time. Taking daily returns, each execution generates a set of 1,231 data (daily returns for each day of the sample period).

For both approaches (described in the sections "a" and "b") the test used to compare the returns is a t-test for normal series and a non-parametric equivalent test for series with no normal distribution (Wilcoxon - Mann – Withney)[8]. A signification level of 95% is applied, unless otherwise mentioned.

c*) Analysis of the distance to the optimum return*

As a complementary approach, an analysis of the distance between the accumulated return obtained by the strategy selected by the GA and the optimum return among those generated by all the possible strategies (which add up to 16,384)[9] in the reference period, is applied. This analysis is appropriate in the sense that if the returns function is very flat, returns very far apart will not present statistical differences, whereas if the returns function is peak-shaped, returns placed very close will present statistical differences. This approach allows simultaneously the study of the optimisation power of the GA methodology.

Sensibility Analysis

1. Interaction between the number of individuals and the number of generations in the execution of the GA.

It seems obvious that an interaction exists between the number of individuals considered in the GA population and the number of generations to be used for the genetic model to converge to optimal values. This interaction is not proportional due to various questions intrinsic to GA functioning, implicit parallelism, minimal deceptive problem, etc.

It is convenient to know if, starting from specific values for both parameters, it is interesting to exchange individuals for a higher number of generations, or vice versa, until the right balance is reached. The interest in carrying out experiments in this direction is based on the objective of limiting the intensive use of computational resources made by the GA, through adequate combinations of both parameters, thus reducing the required time of execution, aspect which is essential in problems where search spaces are very large. If initial population (number of individuals) is excessively reduced, it is probable that, even by increasing the number of generations, it will converge to a local optimum. On the other hand, if initial population is large but the number of generations too reduced, individuals' adaptation process to the environment will be limited.

It is logical to expect that, even if diverse alternatives of the binomial "number of individuals-number of generations" coincide in relation to the total number of different individuals analysed in the GA execution[10], the results obtained will be different, given that individuals' adaptation processes - from those existing in the initial population and the number of generations through which they are allowed to reproduce themselves - are going to be different. However, it is equally reasonable to think that, despite the fact that the GA is going to evolve along different paths for starting from non coinciding initial populations, its functioning, based on the parallel analysis of compact blocks of information and not on individuals themselves, will make it converge, although following different ways, to the same set of solutions. Agrawal and Deb (1999) express their concern about the lack of studies carried out on the subject[11].

The current study suggests two alternatives of parameters for both elements (size of the population and number of generations) with the objective of analysing the sensibility of GA results to different combinations of these parameters. The first alternative uses the values established in section 2, that is to say 50 individuals and 50 generations, while the second one sets the number of individuals at 100 and the number of generations at 25.

Results obtained through tests of accumulated returns, carried out for the five stocks (Ibex-35, Repsol, Telefónica, Endesa and Iberdrola), show that for three of them, Endesa, Repsol and Iberdrola, the use of both sets of parameters does not generate differences in results, since accumulated returns obtained by genetically designed trading systems do not present statistically significant differences. For the other two stocks included in the sample, results are opposite. While in the case of Ibex-35 returns obtained by the trading systems proceeding from the GA version built with 50 individuals and 50 generations are significantly higher in statistic terms than those generated by the version using 100 individuals and 25 generations, it is precisely the opposite which takes place in the case of Telefónica.

Results from tests carried out with daily returns (for the 25 stocks of the sample) do not vary when applying both sets of values for the parameters "number of individuals" and "number of generations", except for two of the stocks: in the case of Santander, daily returns generated by the trading system designed through the GA with 50 individuals and 50 generations are higher than those generated by the GA with 100 individuals and 25 generations while right the opposite happens in the case of Ibex-35.

The analysis made until now do not allow to accept the existence of differences for or against one of the two versions being studied. However, these results could be affected by the type of returns function generated by the set of trading systems. The application of the third type of comparison, analysis of the position of the solution found by the GA in relation to the optimum value, could give more information. This comparison shows how the GA version with 100 individuals and 25 generations obtains higher results than those achieved by the GA version built with 50 individuals and 50 generations. Differences are clear although they are quite small, as shown by the values given in tables 1 and 2.

The obtained results suggest the existence of an interaction between both parameters (number of individuals and number of generations), as well as the robustness of the genetic methodology to the use of alternative combinations of both parameters, within reasonable limits and hypothesis.

Table 1. Highest distance to the optimum return for the returns generated by the trading systems designed by the GA version with 50 individuals and 50 generations

% of the sample[12]	**Distance**	**Stocks**[13]
29%	0,2%	BBV - DRC - ELE - REP - SAN - UNF - VIS
44%	0,5%	CAN - ENC - ERZ - TAB -
60%	1%	BKT - CUB - IGE -MVC -
76%	1.5%	ACX - GES - IBE – SEV
88%	2%	POP - TEF - VAL
96%	2,6%	ACE – FCC
100%	3,4%	IBEX35

Table 2. Highest distance to the optimum return for the returns generated by the trading systems designed by the GA version with 100 individuals and 25 generations

% of the sample[14]	**Distance**	**Stocks**[15]
16%	0,05%	GES - IBEX35 - IGE – TAB
32%	0,1%	DRC - IBE – MVC – VIS
44%	0,2%	ERZ - TEF – VAL
64%	0,5%	ACX - BBV - CUB - ENC – UNF
80%	1%	ELE - POP - SAN – SEV
96%	1,5%	ACE - BKT - FCC - REP
100%	2,12%	CAN

2. Sensibility to the reproduction operator (lineal scaling of the fitness function)

During the initial phase of the GA execution, it is frequent that exceptionally good potential solutions arise, in relation to the rest of the solutions present in the same generation. If their reproduction is not regulated, given the strong weight of their “fitness” function over that of the total population, these solutions would greatly reproduce themselves and could lead to a problem of premature convergence to a local optimum. Scaling of the “fitness” function is one of the available tools, in the sense that it enables to direct the reproduction of potential solutions, thus impeding the best ones to excessively reproduce themselves or the worst ones to prematurely be lost in the first generations of the GA. At the same time, scaling potentiates the differentiation between good solutions in the final phases of the GA functioning, thus reproducing the best ones more than what would occur in accordance with the value of their “fitness” function.

Literature on GA shows that in reduced populations, where the number of individuals is comprehended between 50 and 100, adequate scaling is situated between 1.2 and 2 copies for the individual or potential solution with the highest value in its "fitness" function. Goldberg (1989) points out "Regulation of the number of copies is especially important in small population genetic algorithms".

In order to check the utility of this tool, a third GA version is used, which introduces scaling in the fitness function of the GA described in section 2. This scaling, following the previous recommendation, reproduces twice the best solution of each generation and in a lower proportion the rest of the solutions, always maintaining invariable the average "fitness" function of the generation.

Results of the tests on accumulated returns (applied to the previously mentioned sample, made of five stocks and indexes), show that there are no statistically significant differences between returns generated by trading systems designed through the GA without scaling and the scaled one. Only in the case of Repsol stocks, are results of the GA higher without scaling.

As for tests related to daily returns (applied to the sample consisting of 25 stocks), they show that no statistically significant difference exists between results obtained by both models.

Table 3. Highest distance to the optimum return for the returns generated by the trading systems designed by the GA version with scaling of the fitness function

% of the sample[16]	Distance	Stocks[17]
20%	0.05%	BKT - DRC - FCC - POP – VAL
36%	0.1%	CUB - IBEX35 – IGE – MVC
52%	0.2%	ACE - GES - UNF -VIS
72%	0.5%	ACX – ELE - ENC - ERZ – REP
88%	1%	BBV – IBE - TAB - TEF
96%	1.25%	CAN - SAN
100%	3.5%	SEV

The analysis of the distance to the optimum reveals that introducing scaling in the GA produces higher results than those generated by the same GA without scaling (compare table 3 with table 1). However, differences carry on being rather small.

To complete this study, the effect of introducing the same scaling in the fitness function of the GA built with 100 individuals and 25 generations is also analyzed. Once again, results of the tests on accumulated returns show that no statistically significant difference exists between returns generated by the trading systems designed through both GA models (with and without scaling). Likewise in this case, tests in relation to daily returns do not support any of the two versions (with and without scaling), since differences are not statistically significant, except for one of the 25 stocks making up the sample, Ibex-35, where slightly higher returns are obtained through the model without scaling. Finally, the analysis of the distance to the optimum does not enable to select any of the versions as superior either (see tables 2 and 4). In the case of the GA without scaling, generated returns are situated among 0.1% of the highest returns for 32% of the stocks vs. 24% in the case of the GA with scaling. However, 80% of the stocks, when the scaled GA is applied, generate returns within 0.5% of the highest returns, while only 64% of the stocks generate returns within this range when the GA without scaling is applied.

Table 4. Highest distance to the optimum return for the returns generated by the trading systems designed by the GA version with scaling of the fitness function and 100 individuals and 25 generations

% of the sample[18]	Distance	Stocks[19]
8%	0.05%	REP – VIS
24%	0.1%	BBV - DRC - SAN - VAL
40%	0.2%	ENC - IGE - TEF - UNF
80%	0.5%	ACX - BKT - CAN - ELE - ERZ - FCC - MVC - POP - SEV – TAB
100%	1%	ACE - CUB - GES - IBE - IBEX35

Results shown in this section ratify the robustness of GA methodology. It does not seem necessary to introduce scaling in the fitness function for the GA to converge to optimum solutions. However, it is of some use, albeit very reduced, if the GA incorporates a lower population.

3. Optimisation of crossover and mutation probabilities: a hierarchic GA

Current literature on GA tends to use crossover probabilities situated around 60% - in relation to the number of chromosomes to cross in each generation – as well as very low mutation probabilities, between 0% and 5% - in relation to the number of information bits to mutate in each generation - (Goldberg, 1989; Haupt et al., 1998; De Jong 1975). However, some authors have found that other values work better for these parameters. One of the largest studies carried out on the suitable value of crossover and mutation probability parameters (which also takes into account population size) is the one presented by Schaffer et al. (1989), where 840 different combinations of the previously mentioned parameters are analysed[20]. These parameter combinations were studied for ten different problems, statistically checking "on-line" results – average behaviour of all evaluated individuals. It was found that the most appropriate combinations seemed to be the same ones as those previously recommended by Grefenstette (1986) – using a hierarchic GA, also called meta-GA -: a 95% crossover probability and a mutation probability around 1% (as well as 30 individuals for population size).

The crossover operator leads to the loss of some individuals and the gain of others within a specific generation and, given that it is applied to a population in which the best individuals have previously been potentiated, a crossover probability must be defined to allow the survival of some of these potentiated individuals and, at the same time, to experiment (with the subsequent loss of others) with the creation of new solutions combining the genetic information of those solutions reproduced for their better adaptation to the problem.

The mutation operator, which also leads to the loss of some individuals of the population, to potentiate the appearance of new genetic information – it is indeed the only way to introduce genetic information not represented in the original population – has traditionally been considered as a second rate operator. However, its importance has recently been highlighted in various empirical analyses, among which should be named those carried out by Vose (1994) who found that the introduction of mutations in small quantities can radically change results: "Surprisingly, the addition of small amounts of mutation can radically change outcomes".

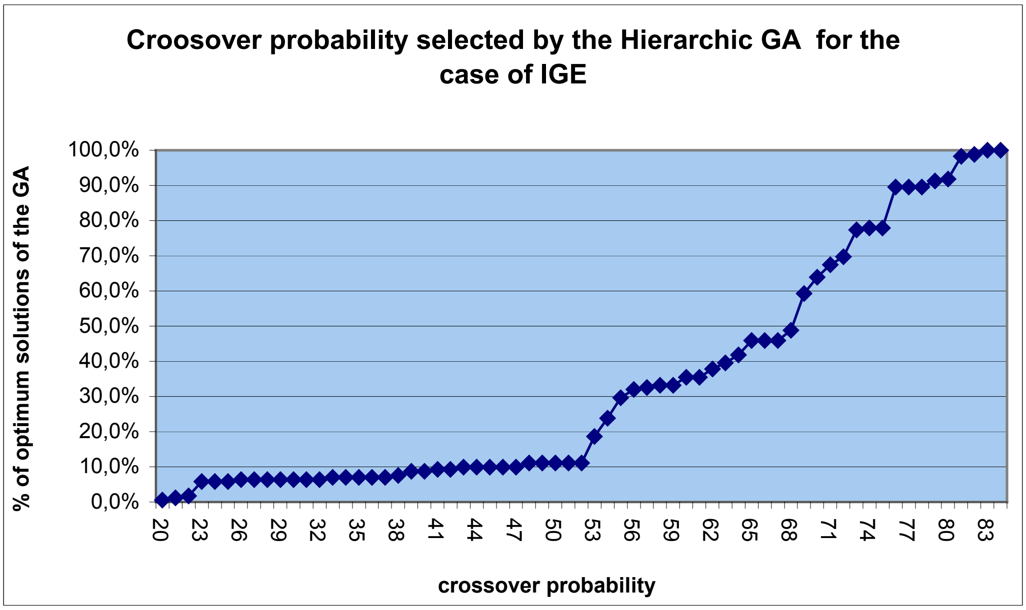
Given that the values established for both probabilities, crossover and mutation, have an incidence on the loss of individuals as well as on the appearance of new ones in each generation, the simultaneous search of both parameters is carried out through the building of a Hierarchic GA. The building of the Hierarchic GA is structured on a *father* GA, which optimises crossover and mutation probabilities that the *son* GA must use, the latter being designed to optimise a simple trading system based on buy and sell signals generated by the crossing of a moving average with the stock quotation. The *son* GA must select the length of the moving average which maximize the accumulated return.

The parameters used in the design of this Hierarchic GA are as follows:

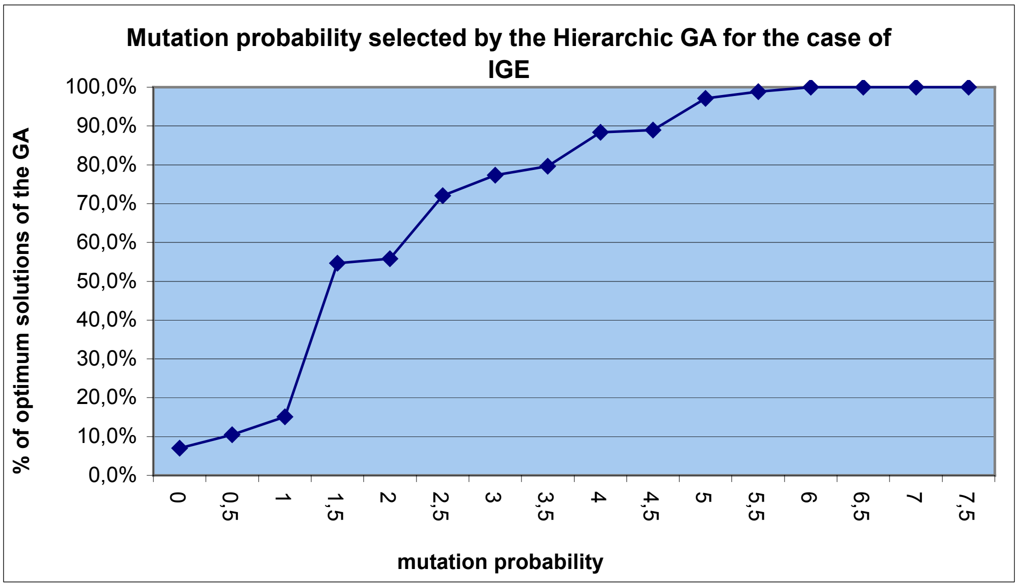
- Number of individuals in the population of the *father* GA: 20 individuals (equivalent to slightly less than 4% of the population)
- Number of individuals in the population of the *son* GA: 12 individuals (equivalent to less than 5% of the population)
- Number of generations considered for the *father* GA: 20
- Number of generations considered for the *son* GA: 10
- Crossover probability for the *father* GA: 55% of the population
- Mutation probability for the *father* GA: 4% of the genes
- Range allowed for the crossover probability of the *son* GA: 20%-84% of the population in 1% brackets (this probability will be optimised by the *father* GA)
- Range allowed for the mutation probability of the *son* GA: from 0% to 7.5% of the genes, in 0.5% brackets (this probability will be optimised by the *father* GA).

Five independent executions of the Hierarchic GA are carried out on each one of the two values which make up the sample of this experiment, that is to say IGE and Ibex-35. A selection is made related to those individuals of the *father* GA – the individuals which make up the population of the *father* GA are possible combinations of mutation and crossover probabilities to apply over the *son* GA – which enable to obtain the best possible solutions with the *son* GA - the best solutions of the *son* GA will be those related to the length of the moving average which generates returns closest to the optimum. Results obtained with the Hierarchic GA are shown in graphics 1 and 2 in the case of IGE and in graphics 3 and 4 in the case of Ibex-35.

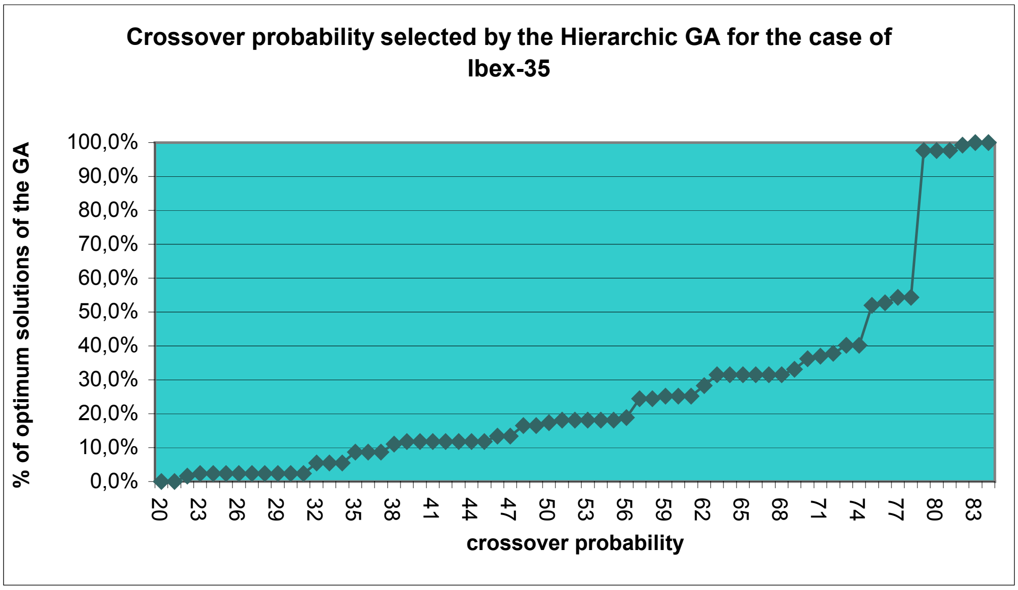
Graphic 1



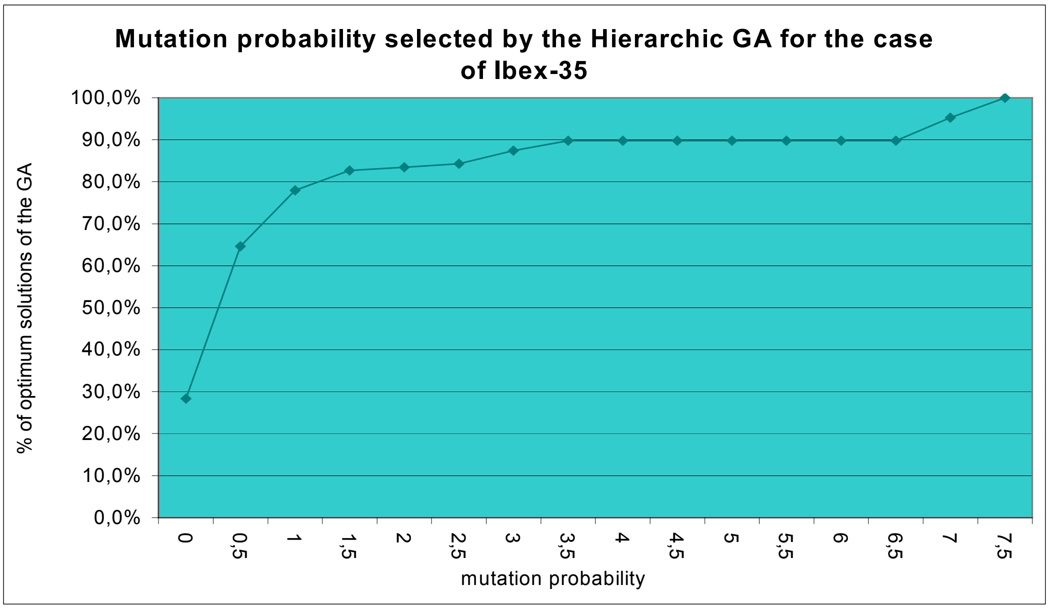
Graphic 2



Graphic 3



Graphic 4



From the previous graphics, it can be deduced that most of the optimum solutions found in the different executions of the Hierarchic GA, both in the case of IGE and Ibex-35, refer to high crossover probabilities of the individuals (within the 54-80% bracket for IGE and 75-80% for Ibex-35) and low mutation probabilities of the genes (specially within the 1-2.5% bracket for IGE and 0-1.5% for Ibex-35).
From the analysis carried out in this section, it can be concluded that the combinations of crossover and mutation probabilities which more often generate optimum results are situated around 75-79% and 0.5%-1.5% respectively[21].

Another important observation, derived from the results obtained in the experiments generated in this section, is the fact that optimum results usually combine high or intermediate crossover probabilities with low mutation probabilities, but also low crossover probabilities with high mutation probabilities. This indicates that a complementarily exists between values of both probabilities (see figure 1).

Figure 1. Optimum solutions: mutation probability break down for crossover probability brackets.

Mutation probability

Crossover probability	**0%**	**3,5%**	**7,5%**
20%	33%	67%	100%
40%	86%	14%	100%
60%	91%	9%	100%
84%			

Utility of "Crossing of Moving Averages" Indicator

Finally and to approach the utility of the charted indicator "crossing of moving averages", results generated by genetically designed CMA systems are compared with data out of the sample. The period considered as excluded of the sample is comprehended between 16-05-1995 and 3-10-95 (100 sessions). The benchmark used to make the comparison is accumulated returns generated over this period by passive investment strategy (buy and hold). Results show that the use of the crossing of moving averages indicator does not enable to obtain better results than those generated by the buy and hold strategy. It is necessary to underline in this point that developed experiments are not designed in accordance with the objective of verifying the utility of the indicator. Therefore, they present some deficiencies when carrying out this checking, such as the fact of not considering the risk associated with operations.

These results seem to be confirmed by literature, as shown by Collin (1994) in the following quote: *"If a rule is easy to find, it will probably appear as a glaring inefficiency in the market and will promptly be arbitraged away, thereby invalidating it. For example, this may be a reason for the poor returns of simple moving average models in the foreign exchange markets over the last 15 years"*. (Colin, "Genetic algorithms for financial modelling" in Deboeck, 1994, p. 149)

The lack of utility of this indicator (CMA) should not, however, lead to reject the set of techniques consisting in technical or charted analysis since *"One can never prove the statement that past return cannot be used to predict future returns since there are an infinite number of ways that the sequence of past prices can be used to forecast futures prices. All that can be done is to test particular ways of combining past price data to predict future returns"*. Elton and Gruber (1984)

Conclusions

The sensibility analysis presented in this study (specifically applied to the design of trading systems) in relation to GA dependence on the value assigned to its control parameters largely confirms the robustness of this technique within some reasonable limits and hypothesis. However, there is an effect, albeit very reduced, of the value taken by parameters on the results generated by the GA. This will have to be considered when analysing the convenience of using or not this technique to solve specific problems.

The details of the results are summarised under the following points:

- The GA seems quite robust to values assigned to the parameters "number of individuals" in the population and "number of generations" considered in the execution, although results show a slight superiority of the GA whose parameters have been defined as 100 individuals and 25 generations over the GA with 50 individuals and 50 generations. In any case, the existence of an interaction between both parameters is clear – which is coherent with the operative mechanism of GA – both having therefore to be set in an overall way.

- The obtained results confirm the robustness of genetic methodology to the introduction or not of some mechanism allowing to somehow direct the reproduction of individuals (as in the analysed case of fitness function scaling). However, the introduction of lineal scaling, which generates two successors for the individual with the best fitness function of each generation, brings slight improvements in the GA operating with 50 individuals and 50 generations. For the GA operating with values established at 100 and 25 for the parameters "number of individuals" and "number of generations" respectively, results do not differ when scaling is introduced. These data confirm the idea expounded by Golberg (1989) in relation to the increased utility of this tool in GA with small populations.

- The overall analysis of crossover and mutation probabilities through the designed Hierarchic GA shows that values which more often lead to optimum results are comprehended in the 75%-79% brackets for crossover probability and in 0.5%-1.5% brackets for the parameter related to mutation probability. On the other hand, a strong complementarily is detected between both parameters, since the GA generates optimum solutions by combining high crossover probabilities with low mutation probabilities, but also (although in less occasions) low crossover probabilities with high mutation probabilities. With regard to future research, it would be interesting to widen the range of mutation probabilities allowed for the *son* GA, which has been situated in 0-7.5% of the genes, to much higher levels in order to define the intensity of the interaction of both parameters.

Finally, the optimisation power offered by the analysed tool - GA - should be underlined. Evidence of it is the observation that the different designed GA always select solutions very close to the optimum. In most cases, these are solutions situated within the 2.5% range of solutions with the best behaviour.

It is necessary to mention the limitations inherent to the analysis carried out, since it is restricted to a short number of experiments. Additional studies would have been necessary to confirm the results obtained here.

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Footnotes

1 The parameters of the corresponding solution can also be directly represented without having to codify them in binary system or by using another type of codification. However, a large number of applications uses this binary codification, maybe because it made up the basis on which mathematical GA fundaments were developed.

2 These stocks are: Acesa, Acerinox, Bankinter, BBV, Cantábrico, Cubiertas, Dragados, Eléctricas Reunidas de Zaragoza, Ence, Endesa, Focsa, Gesa, Iberdrola, Ibex-35, IGE, Metrovacesa, Banco Popular, Repsol, Banco Santander, Sevillana, Tabacalera, Telefónica, Unión FENOSA, Vallehermoso and Viscofan.

3 When both moving averages are built with 1 session, the system follows the strategy of buy and hold. If only one of the moving averages is built with 1 session, the indicator signals come from the crossing of the other moving average with the stock price.

4 See for example Schildt (1992) which is a reference manual of programming language C.

5 Neither the cost of the loan to buy stocks nor the returns generated by selling these stocks are taken into account.

6 The different versions of the GA/refer to the same GA with different control parameters.

7 In order to select 35 trading systems which allow to generate 35 accumulated returns.

8 To test the normality of the series standar tests are used (Lilliefors, Shapiro Wilks and Z. Values).

9 The number of strategies comes from the range of length allowed for both moving averages (1-64 and 1-256). The returns of all strategies are calculated through an enumerative procedure for every stock in the sample.

10 This calculation can only be defined afterwards, since the lower of higher population diversity of the following generations not only depends on the crossover and mutation probabilities being used but also on the fitness structure of the populations belonging to different generations.

11 One of these studies was carried out by Goldberg, who derived a formula to estimate the adequate size of the population depending on the length of the chromosomes being used: population = $1.65.2^{0.21.length}$
This formula estimates at 130;557;2,389 and 10,244 the number of individuals which must make up the population for chains with a length equal to 30;40;50 and 60 respectively.

12 Data contained in each line refer to stocks presented in the actual line plus those presented in the previous ones.

13 Correspondence of the acronyms used for stocks is the following one: Banco BBV (BBV); Dragados (DRC); Endesa (ELE); Repsol (REP); Santander (SAN); Unión Fenosa (UNF); Viscofan (VIS); Cantábrico (CAN); Ence (ENC); Eléctricas Reunidas de Zaragoza (ERZ); Tabacalera (TAB); Bankinter (BKT); Cubiertas (CUB); Indice General (IGE); Metrovacesa (MVC); Acerinox (ACX); Gesa (GES); Iberdrola (IBE); Sevillana (SEV); Banco Popular (POP); Telefónica (TEL); Vallehermoso (VAL); Acesa (ACE); Foccsa (FCC); and Ibex35 (IBEX35).

14 Data contained in each line refer to stocks presented in the actual line plus those presented in the previous ones.

15 The crossing of averages chosen by the GA for the BBV, CAN, IBE, MVC, TAB and TEF stocks are among those existing in the first generation.

16. Data contained in each line refer to stocks presented in the actual line plus those presented in the previous ones.

17 The averages selected by the GA for DRC were already in the population of the first generation.

18 Data contained in each line refer to stocks presented in the actual line plus those presented in the previous ones.

19 The averages selected by the GA for ACE, CAN, IGE, POP and TAB stocks were already in the population of the first generation.

20 The following values were used: 10, 20, 30, 50, 100 and 200 for population size; from 0.05 to 0.95 with 0.10 increases for crossover probability; 0.001, 0.002, 0.005, 0.01, 0.02, 0.05, and 0.10 for mutation probability and two different types of crossover operators (one-point and two-point operator). The computer time (CPU) needed to implement the experiment was 1.5 year and results required a storing capacity close to 100 mb.

21 The complete set of results is available for researchers who are interested (laura.nunez@ie.edu).