

THE PROBLEM OF VARIABLE SELECTION FOR FINANCIAL
DISTRESS: APPLYING GRASP METAHEURISTICS

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Abstract

We use GRASP strategies to solve the problem of selecting financial ratios to model and predict business failure. As a previous step, we use the GRASP procedure to select a subset of financial ratios that are then used to estimate a model of logistic regression to anticipate financial distress on a sample of Spanish firms. The algorithm we suggest is designed “ad-hoc” for this type of variables. Reducing dimensionality has several advantages (Inza et al. 2000) such as reducing the cost of data acquisition, better understanding of the final classification model, and increasing the efficiency and the efficacy. The application of the GRASP procedure to preselect a reduced subset of financial ratios generated better results than those obtained directly by applying a model of logistic regression to the set of the 141 original financial ratios.

Keywords

Financial distress, failure, financial ratios, variable selection, GRASP metaheuristic ks

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1.- INTRODUCTION

From the pioneering works of Beaver (1966) and Altman (1968), many studies have been devoted to the issue of predicting financial distress¹ using accounting based variables. The first studies on insolvency used univariate techniques, Beaver, (1966). Two years later, Altman, (1968) introduced discriminant multivariate analysis which became the predominant technique during the 1970s. Subsequent, in the 1980s, discriminant analysis (whose principle of normality for predictors and equality for variance-covariance matrices is usually violated by the distributions of financial ratios) was complemented by logit and probit analysis, Olson (1980); Zmijewski (1984); Lennox (1999) among others. More recently, researchers have used new approaches to the problem of failure prediction using techniques as neural networks, Altman et al. (1994), genetic algorithms, Varetto (1998), decision trees, Curram (1994), or multidimensional scaling, Neophytou et al. (2004). Examples of empirical analysis on Spanish data are given by Gallego et al. (2002); Laffarga et al. (1990); and Sanchís et al. (2003).

Before performing the discriminant or logit analysis which most business solvency studies are based on, some statistical packages carry out an initial selection of variables in order to eliminate from the analysis the least significant variables. This article addresses such a preselection of variables, which in our case are financial ratios. The search for a variable set is a hard-NP problem and all the feature selection methods used show same drawbacks when dealing with large features sets, as it is the case of financial ratios. Our contribution focuses on designing an ad hoc algorithm that outperforms the “traditional algorithms” currently employed by statistics packages.

Thus, the problem consists in finding a subset of variables that can carry out this classification task in a optimum way. We have to determine the class to which a set of instances belong, characterized by attributes or variables. In supervised learning we have a set of examples characterized by the same attributes as the instances and another attribute corresponding to the class they belong to. Using this set of examples we can create and generalize a rule or set of rules that allows us to classify the instance set with the greatest possible precision.

When dealing with classification problems, the purpose of dimensionality reduction is to eliminate input variables that are not necessary for correct classification. A related research issue is feature selection, which was started in the early 1960s, Lewis (1962) and Sebestyen (1962). According to Liu and Motoda (1998) feature selection has the following purposes: (i) to improve performance (speed of learning, predictive accuracy or simplicity of rules); (ii) to visualize the data for model selection; (iii) and to reduce dimensionality and remove noise. Reducing dimensionality has some advantages such as reducing the costs of data acquisition, better understanding of the final classification model, and an increase in the efficiency and efficacy of such a model. Over the past four decades, extensive research in feature selection has been conducted. Siedlecki and Sklanski (1988) provided a comprehensive review on this

¹ The terms financial distress, insolvency and failure as used in this article refers to both temporary receivership and bankruptcy as defined by Spanish legislation.

subject as early as 1988. Recently, Liu and Motoda (1998) published a book dedicated to feature selection. A lot of works about “feature subset selection” are related with medicine and biology, such as Shy and Suganthan (2003) that investigates feature analysis for the prediction of the secondary structure of protein sequences, Sierra et al. (2001) that predict the conduct of cirrhotic patients, Jaroszewick et al. (2004) with an application in genetic diagnosis of cancer. Another important papers are Tamoto et al. (2004), Lee et al. (2003), Inza et al. (2002), Ganster et al. (2001).

At present the most widely used subset selection technique is the so called “wrapper” approach [Kohavi and John (1997), Jelonck and Stefanowski (1997), Baranauskas and Monard (1998), Sebban and Nock (1999) and Inza et al. (2002)] in which a search algorithm is used to identify candidate subsets and the actual classifier is used as a black box to evaluate the fitness of the subset. Fitness evaluation of the subset however requires crossvalidation or other resampling based procedure for error estimation, requiring the construction of a large number of classifiers for each subset. This significant computational burden makes the wrapper approach impractical when a large number of features are present.

Ideally, we want methods that can guarantee an optimal solution. However, since feature selection is a combinatorial optimization problem, such methods are often computationally infeasible since exhaustive search is required. The most efficient method that can generate an optimal solution is probably the branch and bound algorithm developed by Narendra and Fukunaga (1977). A serious problem, as pointed out by Jain and Zongker (1997), is that the algorithm is still impractical for problems with very large feature sets, as the worst-case complexity of the algorithm is exponential.

Considering that the search for a variable subset is a hard-NP problem, [Kohavi (1995); and Cotta et al. (2004)], metaheuristic techniques can be alternative superior methodologies. These metaheuristic techniques do explorations, searching for those regions where good solutions are located, and then focus the search on those regions. Currently, these techniques are used to solve many types of optimisation problems although originally the majority were designed to solve specific combinatorial optimisation problems. Within this category we can include most problems with a finite number of alternative solutions or at least with numerable alternative solutions. In real-world applications, people are more interested in obtaining good solutions in a reasonable amount of time rather than obsessed with optimal solutions. Therefore, we favor metaheuristic methods that are efficient in dealing with real world applications and obtain reasonably good solutions without having to explore the whole solution space.

Within the metaheuristic strategies applied to the variable selection problem, one of the most used is the Genetic Algorithms technique (GA) [Bala et al. (1996), Jourdan et al. (2001), Oliveira et al. (2003), Inza et al. (2001a, 2001b) and Wong and Nandi. (2004)]. Intuitively, this is a good approach since GA is evolutionary and is supposed to find good solutions quickly by effectively combining high-performance strings. It seems that the only drawback, as noted by Jain and Zongker (1997), is its difficulty in finding the overall best solution, which is not a big concern when dealing with real-world applications. However according with Huang (2003), after conducting several case studies using simple GA for dimensionality

reduction, he found that the approach is not as efficient as he has hoped. For larger problems, premature convergence was observed at around the 10th generation. Although by adjusting the mutation probability, the problem of premature convergence can be partially overcome, a near optimal solution usually takes a long time. Besides, in Ferri et al.(1994) is shown that the performance of GA degrades as the dimensionality increases. It is unclear how key parameters involved in GA can be determined, such as population size, mutation probability, and fitness measure, to achieve its promised efficiency and effectiveness.

In summary, all of the feature selection methods shown before exhibit some drawbacks when dealing with problems with very large feature sets and real-world applications. Therefore, we have decided to develop a new approach emphasizing these points.

In our case, we only use quantitative variables (financial ratios) to carry out the classification of firms into two groups: healthy firms and financial distress firms. The exclusive use of quantitative variables allows better measurement and comparison of their classificatory and discriminant capacity. Thus, we can develop variable selection methods especially adapted to these kinds of variables, which will therefore be more efficient. Specifically, for solving the feature subset selection problem an algorithm based on GRASP (Greedy Randomized Adaptive Search Procedure) strategies is designed. We conclude that our algorithm is more efficient than the selection methods that some well-known statistics software like SPSS and BMDP use.

After describing and checking our GRASP algorithm, it is used for selecting financial ratios on a sample of Spanish companies. Those ratios selected by the GRASP are then used to feed a Logit model, that is called the GRASP-LOGIT model. The results obtained by the GRASP-LOGIT model are superior to those from the traditional logit.

The remainder of this paper is organized as follows. Section 2 provides a description of the GRASP procedure. The sample is described in Section 3. Section 4 shows the results obtained by applying the GRASP metaheuristics to the selection of financial ratios. In Section 5 the results of the estimation of the GRASP-LOGIT model are presented. The last section, Section 6, reports some key conclusions.

2.- DESCRIPTION OF THE GRASP ALGORITHM

2.1.-MODELLING AND FORMULATION OF THE PROBLEM

Let $A = \{ a_1, a_2, \dots, a_n \}$ be a set of n cases or instances and let $V = \{ v_1, v_2, \dots, v_m \}$ be a set with m variables; (in order to simplify, V will be equally identified with the coefficients, i.e., $V = \{ 1, 2, \dots, m \}$). Each instance a_i (each company) is defined as:

$$a_i = (a_{i1}, a_{i2}, \dots, a_{im} | c_i), \quad [1]$$

In other words, each instance is defined by the value the variables take (i.e., the financial ratios) and the class it belongs to (solvent or insolvent).

Given a predefined value $p \in N$ we have to find a subset $S \subset V$, with a size p and the greatest classificatory capacity. In order to measure the classification capacity of the different subsets S , let us consider k partitions previously defined for the set A . In each partition there are 2 subsets, A_1 (training set) and A_2 (validation set). In other words, $A = A_1 \cup A_2$, where A_1 and A_2 have the same proportion of elements of each class as A . The cardinal number for all the subsets A_1 is the same (and therefore, the same applies to A_2). For each subset of variables S , and for each pair of instances a_i and a_t , we define the following distance:

$$d(a_i, a_t) = \sum_{j \in S} d_j^2(a_i, a_t) \quad [2]$$

where

$$d_j(a_i, a_t) = \frac{|a_{ij} - a_{tj}|}{\max_j - \min_j} \quad [3]$$

with \max_j and \min_j being the maximum and minimum values of the variable v_j observed in the training set.

In order to determine the goodness-of-fit $f(S)$ of each subset of variables S we carry out the following process for each partition under consideration: for each instance a_i of the validation set A_2 we determine the closest instance to the training set A_1 , a_{i^*} , and we assign to a_i the class a_{i^*} belongs to. The percentage of total hits is the goodness-of-fit $f(S)$ of each subset S .

2.2.- DESCRIPTION OF THE GRASP ALGORITHM

Our method is based on constructive GRASP. GRASP, or Greedy Randomized Adaptive Search Procedure, is a metaheuristic strategy that builds up solutions by using controlled randomness with a greedy function. Most GRASP implementations also include local search which is used to improve the solutions generated by the greedy-random method. This is also the case in this paper. GRASP was originally suggested for the set covering problem, Feo and Resende (1989). Details of such a methodology and its most recent applications can be found in Feo and Resende (1995) and Pitsoulis and Resende (2002).

The operating scheme of our GRASP algorithm is as follows:

Repeat

Build a solution by the greedy-random method

Improve the solution by local search

Update the best solution obtained to that moment

till a stop criterion is satisfied

The stop criterion is satisfied when a preset number of iterations (max_iter) takes place without improvement. The two main procedures are described below: the greedy-random method and local search.

2.2.1.- The greedy random procedure

The greedy function guiding the entry of variables into the solution is based on very well-known results over variance decomposition. In more specific terms, let x be any variable defined on the n cases under consideration, that is, $x' = (x_1, x_2, x_3, \dots, x_n)$, ng is the number of classes and nn_i is the number of cases of the group i , $i = 1 \dots ng$. In addition:

\bar{x} : mean of the variable x in the set of n cases;

\bar{x}_i : mean of the variable x in the cases of the class i ; $i = 1 \dots ng$;

$cl(j)$: which is the class the individual j belongs to.

We define:

$$VT(x) = \sum_{j=1}^n (x_j - \bar{x})^2 \quad (\text{total variability}) \quad [4]$$

$$VE(x) = \sum_{i=1}^{ng} nn_i (\bar{x}_i - \bar{x})^2 \quad (\text{between-group variability}) \quad [5]$$

$$VI(x) = \sum_{j=1}^n (x_j - \bar{x}_{cl(j)})^2 \quad (\text{in-group variability}) \quad [6]$$

and $F(x) = \frac{VE(x)}{VI(x)}$. [7]

It is known that $VT(x) = VE(x) + VI(x)$. We also know that the function $F(x)$ is a good measure of the discriminant capacity of each variable.

Let S be the solution that is going to be built; the greedy-random procedure is described of the following way:

1. *Start: Make $S = \emptyset$*
2. *Calculate $F_j = F(v_j)$, $j = 1 \dots m$*
3. *Determine $F_{max} = \max \{F_j/j = 1..m\}$ and $F_{min} = \min \{F_j/j = 1..m\}$*

4. Build $L = \{ j/F_j \geq \alpha F_{max} + (1-\alpha) \cdot F_{min} \}$
5. Select $j^* \in L$ randomly and make $S = \{j^*\}$
6. While $|S| < p$ make:
 - a. Let $S = \{ j_1, j_2, \dots, j_t \}$ (the variables which are already in the solution)
 - $\forall j \notin S$: - Determine the values of the variable r_j in the following linear model by ordinary least square

$$v_j = \alpha + \beta_1 \cdot v_{j_1} + \beta_2 \cdot v_{j_2} + \dots + \beta_t \cdot v_{j_t} + r_j$$
 - Calculate $F_j = F(r_j)$
 - b. Determine $F_{max} = \max \{ F_j/j \notin S \}$ and $F_{min} = \min \{ F_j/j \notin S \}$
 - c. Build $L = \{ j/F_j \geq \alpha F_{max} + (1-\alpha) \cdot F_{min} \}$
 - d. $j^* \in L$ randomly and make $S = S \cup \{j^*\}$

Thus, the F function previously defined, is the guide in the variable selection procedure. However, we do not necessarily choose at each step the variable corresponding to the highest value of F , F_{max} . In such a case we build the set L (called “the candidate list”), which is made of those variables with the highest values and one is randomly chosen from the list.

Initially, the guide function is the value of the function F in the original variables. Later, we use the F value, not in the original candidate variables to entry into the solution, but in the residues that are obtained when we remove from such variables the information already provided by the variables in solution S . This concept is used by some statistical software applications such as BMDP and SPSS in the variable selection procedure which they run prior to executing the true discriminant techniques. The procedure used by these statistical softwares (BMDP and SPSS) differs from our GRASP method in that their variable selection are deterministic and the variable selected always corresponds to F_{max} , while our GRASP procedure introduces some randomness. One of the specific advantages of the greedy random method is that the best solution obtained by repeating this procedure tends to be better than the one obtained by deterministic selection. This is also the case in our study, as we show in the following sections.

The α parameter is used to control the degree of randomness of the procedure. The greater the value of α , the lower the degree of randomness. If $\alpha = 0$, the procedure is totally random, because L or the “candidate list”, would be made up of all the variables not included in the solution. If $\alpha = 1$ L would only be made of the variable corresponding to F_{max} . From now on we will denominate the method suggested when $\alpha = 1$ as constructive deterministic.

2.2.2.- The Local Search Procedure

Each complete solution S generated by the greedy-random procedure is improved by a simple local search procedure. In each local search step a variable in the solution will be exchanged for another outside the solution. In more specific terms, let S be a solution, and we define

$$N(S) = \{ S'/S' = S \cup \{j'\} - \{j\}, \forall j \in S, j' \notin S \} \quad [8]$$

The local search procedure can be described as follows:

Read initial Solution S

Repeat

Make previous_value = f(S)

Search f(S) = max { f(S')/S' ∈ N(S) }*

If f(S) > f(S) then make S = S**

till f(S) ≤ previous_value*

Thus, the procedure ends when no exchange provides a better solution.

3. SAMPLE SELECTION AND FINANCIAL RATIOS

3.1 COMPANIES

The sample consists of 198 Spanish companies of which approximately one-third, (67), were failed companies placed under temporary receivership or declared bankrupt in 2003². The other remaining companies, (131), were healthy, or at least “active” firms. The companies were selected from the SABI database from Bureau Van Dijk (BVD), one of Europe's leading publishers of electronic business information databases. BVD is best known for its range of financial information products being one of the providers of Wharton Research Data Services. BVD Databases has been used in previous failure studies on companies from European countries [i.e. Ooghe et al. (2002)]. SABI comprises all the companies whose accounts are placed in the Spanish Mercantile Registry. The firm's selection was made randomly for each group (failed/healthy), but only choosing from limited liability companies and corporations. Only those with complete (or almost complete) data available for the three previous years were included³. Therefore, our sample selection method do not follow the usual paired sample by sector and size. Not all authors follow such paired sampling due to its arbitrariness and the lack of empirical evidence to support or reject the superiority of such a procedure [see Ohlson (1980: p. 112)]. It could be actually more interesting to include the variables size and sector as predictors, than use them for matching [see Lennox (1999)].

² Out of these 67 companies from the sample, 18 (27%) were placed in temporary receivership, whereas 49 (73%) were declared bankrupt.

³ The rate of unavailable values, (244), for the data set gathered – which was 27,918 – was below 1%. In these cases, data from the previous year was used, or if that was also unavailable, we used the next period.

The majority of companies that failed in 2003 had no data available for 2002. Thus, the sample selection criteria was based on the availability of data for 3 consecutive years, i.e., either 2002, 2001, and 2000 or 2001, 2000, and 1999. This factor introduces some bias, because all healthy companies – that are “active” at least until December 2003 - had data available for 2002, 2001 and 2000, while on the other hand, only seven companies in the failed group had data available for those years; thus we had to use data from 2001, 2000, and 1999 for the remaining ones. However, this bias is ameliorated to a great extent by the fact that “active” status refers to December 2003, whereas insolvent business status refers to any time in 2003; in fact, 67% of the companies became insolvent in the first half of 2003, and 100% in the first 9 months.

Table 1 shows the data for both distributions (failed/healthy) by sectors. Although the samples have been selected in a random way, without taking into account the sector the companies belong to, it is interesting to notice that 55 of the 67 failed firms belong to the same sector that healthy firms according to the two digit CNAE (Spanish Classification of Economic Activities) code.

<Table 1 about here>

Table 2 shows the distribution mean by size (measured by the number of employees) and age, and the proportion of firms in both legal structures (corporation / limited liability company). As expected, the mean size of solvent companies was greater than that of the insolvent companies. However, by taking away from the sample those solvent companies with more than 100 employees (only 10 of them in total) their mean size was reduced to the point of the insolvent group. It is also interesting to note that the legal structure of the companies is equally distributed in both groups, with 60% being limited liability companies and the remaining 40% corporations. On the other hand, it is surprising that the mean number of operating years for both groups of companies is the same, 18 years, with a very similar standard deviation. It is usually argued that most failures takes place in the first years of the company existence. In this sense, our analysis includes a survival bias which might partially explain these data. This is so because although the sample was selected randomly, we have to impose the condition of having data available for the 3 years preceding 2003 (solvent companies) or 2002, if no data was available for 2002, which often happened with the insolvent group. Our data on operating years seems to indicate that once companies operate for more than 2 or 3 years, the probability of becoming insolvent is not related to their years in business.

<Table 2 about here>

3.2 FINANCIAL RATIOS.

Thirty-six ratios out of those published in the SABI database were selected for each company for each of the 3 years preceding 2003, or 2002 when applicable. This yielded a total of 108 data per company. All the ratios published in SABI for the Spanish companies were effectively included, except for a few for which there was no consistent information available, as was the case for the ratio “credit period”, which unfortunately had to be excluded. On the

other hand, 11 new ratios were added which referred to time trend for 11 of the 36 ratios previously selected: three time trend were calculated for each ratio - trend between year t and $t-1$, between $t-1$ and $t-2$, and between t and $t-2$. Therefore, the total data for each company was 141 (108 plus 33). Including time variations for the ratios is not a common practice in insolvency analysis, with the exception of some few papers as the conducted by Becchetti et al. (2003). However, this can be of great interest as it is well known that the ratio distribution in healthy companies tends to be constant over time, whereas it varies greatly in insolvent companies due to ratio deterioration, [e.g. see Beaver (1966)]. Bearing in mind this factor, time variations in some ratios could have a greater predictive power than the own ratio value. On the other hand, it seems a priori that such variations might have greater independence from the activity sector and company size than the own ratio. Tables 3a and 3b show the definition of the financial ratios set and their main descriptors, respectively

<Table 3 about here>

The relationship between the mean values of the ratios in both groups generally is the expected one, with some exceptions (financial costs %, liquidity ratios, etc.). However, when such exceptions are examined in detail, we see that they are due to extreme values in the ratios of some of the companies.

4.- APPLYING GRASP AS RATIO PRESELECTION PROCEDURE

4.1.- PREVIOUS COMPUTATIONAL EXPERIMENTS

In order to compare the efficiency of our GRASP algorithm and its components, we carried out some tests as a previous step. We used the table of 141 financial ratios for a total of 198 companies. From this table we obtained smaller tables with an m number of financial ratios for the 198 companies. Thus, we consider the following values of m , $m = 40$ (corresponding to the first 40 financial ratios), 65, 90, 105, and 120.

The number of cases (companies) under consideration is 198, divided into classes (healthy and failed), with 131 and 67 items, respectively. We consider a partition, randomly obtained, $A = A_1 \cup A_2$, where A_1 has 100 items (66 solvent and 34 insolvent) and A_2 has 98 (65 and 33).

Table 4 shows the results obtained, expressed as percentage of hits, for the constructive deterministic algorithm (the one used by software packages like BMDP and SPSS), for 20 executions of the greedy-random method⁴ ($\alpha = 0.85$), and for our GRASP procedure⁵ ($\alpha =$

⁴ Which consist of the introduction of some randomness in the constructive algorithm.

⁵ Which introduces a local search procedure over the preceding.

0.85 and $max_iter = 20$), for different values of p (number of ratios selected) and different values of m (number of ratios under consideration).

<Table 4 about here>

Table 4 shows that the repetition of the greedy-random method gives better results than the constructive deterministic method: in 17 cases it is better (in bold), in seven the same and only in one case is it worse. The GRASP method (which includes the greedy-random method and local search) strongly improves the results of the greedy-random method on its own. Therefore, local search is very efficient for improving the quality of the solutions obtained by the different constructive algorithms.

4.2.- PRESELECTION OF RATIOS BY GRASP

In this section, we solve the problem of variable selection for our sample now that the efficiency of the GRASP algorithm has been demonstrated. As previously stated, we deal with 198 cases (firms), divided into two classes (healthy and failed), with 131 and 67 items, respectively. We consider the same partition as in previous tests, $A = A_1 \cup A_2$, where A_1 has 100 items (66 healthy and 34 failed) and A_2 has 98 (65 and 33). In this case we use the total number of variables or ratios ($m=141$).

Table 5 shows the values of the objective function obtained for the different values of p ($p=10, \dots, 15$). In each column the result for one of the three strategies used is shown: *constructive deterministic*, *constructive greedy random* (executed 20 times and $\alpha = 0.85$), and *GRASP methods* ($\alpha = 0.85$ and $max_iter = 20$).

<Table 5 about here>

For each value of p the value of the objective function (the percentage of hits), is better when the GRASP procedure is applied (third column in the table). This makes sense because in the GRASP procedure the solution obtained is improved by applying local search. Therefore, this metaheuristic strategy provides us with the best solutions. On the other hand, the greedy random constructive method (2nd column) generates better results than the constructive deterministic strategy which coincides with the selection method used by statistics software like SPSS and BMDP. This means that the results obtained by these statistical packages can be improved simply by adding randomness to the constructive deterministic method or by a more complex metaheuristic strategy, such as GRASP; this will improve the solutions obtained with the random constructive method by applying a local search procedure.

The number of financial ratios allowed for selection (p), ranges between 10 and 15. Note that if this number increases, the value of the objective function does not necessarily increase. In any case, the best values for the objective function are obtained when $p=13$, $p=14$, and $p=15$ with the GRASP procedure. It is also interesting to see that the value of f for $p=10$ is 0.71020408 when using the greedy random constructive method, but when $p=11$, $f =$

0.70408163 which means that the percentage of hits is lower in spite of having increased the value of p .

Finally, Table 6 shows the frequency of selection for the ratios. Columns 2, 3 and 4 show the number of times each financial ratio is selected by the different strategies used: constructive deterministic, greedy random constructive and GRASP. The first column shows the total number of times such a ratio has been selected by the set of strategies and the last column shows the kind of ratio it is: A (activity), R (returns), E (equilibrium), S (solvency), L (liquidity), E_C (equilibrium_cinetic), and PE (per employee).

<Table 6 about here>

If we focus on the financial ratios selected, that is, those ratios that can better predict corporate failure we can conclude the following:

- Normally, the ratios more often selected are those referring to activity, solvency, and to a lesser degree, return. In more specific terms, the most relevant ratios are: Added Value Growth, Solvency ratio, Productivity, ROA before taxes, and Equity over Permanent Funds. As a whole, these financial ratios enable us to obtain good knowledge regarding the solvency of the company. However, it is interesting to point out that the “leading” ratios are not always the same in each selection procedure, as shown in Table 6.
- On the other hand, ratios referring to trends (time variations) are the most prominent type within the selected ratios, either between year t and $t-1$ or t and $t-2$ and $t-1$, $t-2$ ⁶. Eighteen models have been tested: 6 models (with values of p ranging from 10 to 15) for each of the three strategies under consideration (constructive deterministic, greedy random constructive, and GRASP). In 16 out of the 18 models at least one trend ratio is always selected. Therefore, although trend ratios are not usually included in this kind of analysis, they are important. The relevance of time variability in financial ratios dealing with solvency and debts, which are the ones with the highest frequency in all the models tested, makes sense because the worsening of these ratios over time might suggest that the company is close to an insolvency situation. From its beginning, the literature on financial distress [see Beaver (1966)], suggests that the ratio distribution of healthy companies is steady over time whereas it changes in a significant way for unsound companies.

5.- A “GRASP-LOGIT” MODEL

Finally, in order to perform a whole analysis, besides solving the problem of variable selection, we have made use of logistic regression to fine tune the ratios that best predict the insolvency situation of a company. To this end, we took the selected ratios with the best value for the objective function (shown in bold in Table 5), which corresponds to the GRASP

⁶ For reasons of space this information has not been included in Table 6. It is available upon request.

metaheuristic strategy when $p=14$ ($f=0.82040816$). In this case, the 14 variables selected to which we apply logistic regression are shown in Table 7.

<Table 7 about here>

After performing the logistic regression, the financial ratios that best predict corporate failure out of the 14 ratios in Table 7 are:

- ROA before taxes_t
- Solvency ratio_t-1
- Value Added Growth_t-1

The global percentage of hits in this analysis is 78.8% for a cut-off probability of 0.5. Although we cannot base our analysis on the hits in each group (healthy/failed) because type I and II errors have not been taken into account in the ratio preselection process using GRASP, we have performed different cutting points, in order to balance both types of errors, getting a similar global fitness. For instance, a cut-off point of 0.67 results in a global fitness of 77.8%, with fitness for type I and II errors of 76.2% and 78.6% respectively.

We have introduced control variables for the size of the company (measured by the Number of Employees) for the age and for the sector they belong to (using National Classification of Industry Activities CNAE-1 digit) in the GRASP-LOGIT model. However, these variables had no effect on the final results of the model. Neither the size of the company, nor the age⁷, nor the sector it belongs to, seem to have any predictive value regarding insolvency.

The result obtained makes sense because it uses three of the key variables in the financial analysis of the company. On the one hand, these identify the business return (ROA before taxes_t) and its recent evolution (Added Value Growth_t-1) and on the other, the leverage of the company (solvency ratio_t-1). Besides, these ratios are not biased by the activity sector which the firm belongs to.

- ROA shows the capacity of the company to obtain returns from its assets and to some extent this variable is immune to what sector the company belongs to. In the well known "Du Pont" analysis, ROA is decomposed into sales margins and total turnover of assets as indicated in the following expression:

$$ROA_{\text{before taxes}} = \frac{\text{Profit}_{\text{before taxes}}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Total Asset}}$$

Normally, those sectors which are capital-intensive have a lower asset turnover (they have greater fixed assets) than those with a lower need for capital because their

⁷ It is necessary to point out that there is an important bias in the analysis of the age, as the sample selection was made of firms with at least three years of life.

investment needs in assets are lower. However, capital-intensive sectors have a greater sales margin than those which are less capital-intensive. Therefore, given that ROA takes both variables together, this palliates, to a great extent, the effect of belonging to one sector or another.

- The solvency ratio represents the equity-debt level of the company, and by combining this with ROA before taxes, using Du Pont's analysis decomposition ratios, we obtain ROE before taxes, as shown in the following expressions:

$ROE_before_taxes = ROA_before_taxes \times leverage$

$$ROE_b_taxes = \frac{\text{Profit_before_taxes}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Total_Asset}} \times \frac{1}{\frac{\text{Total_Equity}}{\text{Total_Asset}}}$$

- Finally, the Added Value Growth shows the time evolution of the operating profit. Thus, given the level of solvency in the firm, a positive value of this rate would involve, in principle, an improvement in the financial situation of the company, and a negative value, the worsening of its financial situation.

Therefore, we can conclude that ROE (through ROA and Leverage) together with Added Value Growth are key ratios to forecast financial distress.

To make sure that the model forecastability is not the result of overfitting, we have tested our GRASP-LOGIT model with out-of-sample data using 61 companies (of which 40 are healthy and 21 failed firms) selected randomly from each group. The global fitness obtained with out-of-sample data is 77.04% comparing with the 78.8% with in-sample data, which confirms the forecastability of the model.⁸

Finally, in order to analyse the advantages of the GRASP method for solving this problem of variable selection before applying logistic regression (and the deterministic constructive algorithm that SPSS uses), we have also carried out a logistic regression on the 141 original variables so that we can make comparisons. These are the results:

- The percentage of global hits is very similar to the one obtained for the 14 variables preselected by GRASP (79.3% compared to 78.8% for GRASP-LOGIT). This is so, despite the greatest number of variables included in this new model comparing to the GRASP-LOGIT. The 79.3% is obtained by a model with 7 variables, while the GRASP-LOGIT rises to 78.8% using only 3 variables (less than half). Obviously, this is due to the good performance of the GRASP algorithm.

⁸ We check again that type I and II errors can be balanced changing the cutting point, maintaining the same level for the global fitness.

- The selected variables in the logit with 141 variables are the following:
 - Value Added Growth $_t$ (%)
 - Value Added Growth $_{t-1}$ (%)
 - Productivity $_t$
 - Equity over permanent funds $_t$ (%)
 - Debts $_{t-2}$ (%)
 - ROA before taxes $_t$ (%)
 - Personnel expenditures $_{t-1}$ (%)

Within the seven variables selected in this case — or six if we do not take into account the time factor — we find the three variables which were previously selected by the GRASP-LOGIT model (ROA before taxes $_t$, Added Value Growth $_{t-1}$, and Debts $_{t-2}$). Debt is a variable equivalent to the variable solvency ratio that appeared in the GRASP-LOGIT (although its reading is the opposite) because:

$$\text{Solvency ratio} = 100 - \text{Debts}$$

This latter variable now appears in the $t-2$ period, while in the first GRASP-LOGIT model, the solvency ratio appeared in the $t-1$ period.

The remaining selected variables for this model are personnel expenditures (%), productivity (gross operating margins per monetary unit used in labour), and equity over permanent funds. The meaning of these variables as predictors of failure is not as clear as for the three variables obtained with the GRASP-LOGIT model. Personnel expenditures (measured as a percentage of the firm income) show great dependency on sector, because the more labor-intensive sectors show higher values for this variable. The opposite happens with productivity; i.e., the sector which is most labor intensive has lower figures for this indicator. Finally, the variable equity over permanent funds or long-term funds does not seem to be a good predictor of insolvency, because it does not take into account short-term debts, which in many cases can be decisive for assessing the payment capacities of the company.

Therefore, it seems that the interpretation that can be derived for the results obtained by the GRASP-LOGIT model is better than the ones from the logit with 141 ratios, whereas the predictive capacity of both models is the same.

7.- CONCLUSIONS

This work has focused on the resolution of the problem of financial ratio preselection to model business insolvency -we use 141 financial ratios over a sample of 198 Spanish firms. To this end we used the metaheuristic strategy GRASP (Greedy Randomized Adaptive Search Procedure) which builds solutions by controlled randomness over a greedy function which guides the entry of variables into the solution. Then, the variable selection is improved by local search. This strategy can be used for solving the feature subset selection problem when

all the variables are quantitative. There aren't any references in the literature about algorithms designed "ad hoc" for this type of variables.

The results obtained with GRASP and its elements were compared to those obtained by applying the deterministic constructive algorithm used in statistical software, such as BMDP and SPSS. The systematic superiority of GRASP means that the quality of the solutions found can be improved by introducing randomness in the selection procedure or by using local search.

In addition, we modelled business insolvency by applying a logistic regression model to the results from the GRASP procedure. GRASP was used to preselect 14 financial ratios from which the logit was built. We called this model GRASP-LOGIT, and the results obtained with it were compared to those obtained by applying a logit directly to the original 141 financial ratios. Although the classificatory capacity of the GRASP-LOGIT is the same as the logit model with 141 ratios, the explanatory capacity and the simplicity of the former is greater than the latter. Therefore, we can assert that incorporating the GRASP metaheuristic into the preselection of financial ratios adds an improvement to the understanding of business insolvency model. Its advantage is also apparent concerning reducing the cost of data acquisition.

The GRASP-LOGIT model shows that the best combination of ratios to explain corporate failure are: ROA before taxes, Solvency ratio, and Added Value Growth. The first two ratios are the components of ROE identified by Du Pont's analysis. Besides, our results reveal that neither the size of the company (measured by the number of employees), nor the age, nor the sector it belongs to seem to have any predictive value regarding modelling insolvency.

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TABLES

Table 1 Breakdown of failed / healthy firms by CNAE classification.

CNAE	Failed	Healthy
01 Farming	0	3
02 Forestry	0	1
15 Food and beverage sector	5	6
17 Textile industry	3	1
18 Clothes industry	1	1
19 Shoemaking	0	1
20 Wood and cork industry	1	2
21 Paper industry	0	1
22 Publishing and graphic arts	2	3
24 Chemical industry	0	4
25 Manufacturing of plastic and rubber products	1	2
26 Manufacturing of other mineral products	0	1
27 Metalwork	2	1
28 Manufacturing of metal products	4	3
29 Building machinery	5	3
31 Manufacturing of electric equipment	2	0
33 Manufacturing of medical equipment	0	1
34 Manufacturing of motorised vehicles	0	1
35 Manufacturing of other transport material	1	0
36 Manufacturing of furniture; other industries	4	3
41 Water collecting, purifying and distribution	0	1
45 Building	10	16
50 Sales and repair of. motorised vehicles	0	5
51 Wholesale sales	12	16
52 Retail sales	7	11
55 Hospitality sector	0	4
60 Land transport	0	2
61 Sea transport	0	1
63 Transport-related activities	1	1
65 Finance trading (except insurance)	0	1
70 Estate agents	2	16
74 Other business activities	2	12
80 Education	2	0
85 Hospital and veterinary activities	0	2
92 Cultural, recreational and sport activities	0	4
93 Personal services activities	0	1
Total	67	131

Table 2 Mean size, Legal format and Years in business

	Insolvent (67)	Solvent (131)
Size*		
Mean number of employees	22 (22)	36 (65)
Mean number of employees (<100 employees)**	22 (22)	20 (23)
Legal format***		
Corporation	27 (40%)	52 (40%)
Limited Liability Company	40 (60%)	79 (60%)
Years in business*		
Mean number of years in business	18 (15)	18 (13)

* Standard deviation in brackets

** After eliminating from the sample those solvent companies with more than 100 employees (a total of 10)

*** Number of companies (percentage in each sample in parenthesis)

Table 3a Ratios definitions

Activity Ratios.	
Sales growth (%)	$[(\text{Sales}_t - \text{Sales}_{t-1}) / \text{Sales}_{t-1}] \times 100\%$
Asset turnover	Sales/Total Assets
Productivity	$[\text{Operating revenues} - \text{Consumption and Operating expenditures}] / \text{Personnel expenditures}$
Personnel expenditures (%)	$[\text{Personnel expenditures} / \text{Operating revenues}] \times 100\%$
Value added growth (%)	$[(\text{Value Added}_t - \text{Value Added}_{t-1}) / \text{Value Added}_{t-1}] \times 100\%$
Operating margin (%)	$[\text{Earnings before Taxes} / \text{Operating revenues}] \times 100\%$
Net Asset Turnover	Operating revenues/Permanent funds
Return Ratios.	
ROCE	$[\text{Earnings before Taxes} + \text{Financial expenses}] / \text{Permanent funds} \times 100\%$
ROA	$[\text{Earnings} / \text{Total assets}] \times 100\%$
ROA before taxes	$[\text{Earnings before Taxes} / \text{Total assets}] \times 100\%$
ROE	$[\text{Earnings} / \text{Equity}] \times 100\%$
ROE before taxes	$[\text{Earnings before Taxes} / \text{Equity}] \times 100\%$
Financing costs (%)	$[\text{Financing costs} / \text{Sales}] \times 100\%$
Equilibrium Ratios.	
Working capital (€)	Equity + Provisions for C & E+ LT Creditors – Fixed assets
Need for Working capital (€)	$[\text{EHNDP} + \text{Accrued Expenses} + (\text{Inventory} + \text{Accounts Receivable})] -$ $[\text{Accrued Incomes} + \text{Accounts Payable}]$

Cash (€)	ST Financial Investments + Cash – ST Debt
Equilibrium	Equity + R & C Provisions for C & E+ LT Debt] / Fixed assets
Cinetic Equilibrium Ratios.	
Working capital (days)	[Working capital / sales] x 360
Need for Working capital (days)	[Need for Working capital / Sales] x 360
Cash (days)	[Cash / Sales] x 360
Clients' Credits (days)	[Accounts Receivable / Operating Incomes] x 360
Clients' Credits due to Sales (days)	[Accounts Receivable / Sales] x 360
Solvency Ratios.	
Debt (%)	[Total liabilities / Total liabilities and Owners' Equity] x 100%
Solvency Ratio (%)	[Equity / Total assets] x 100%
Equity over Permanent funds (%)	[[Equity / [Equity + LT creditors + Provisions for C & E]] x 100%
Repayment capabilities	[LT and ST creditors / [Sales + Depreciations + Provisions + Equity]
Liquidity Ratios	
Immediate Liquidity	[ST Financial Investments + Cash] / Accounts Payable]
Current Liquidity	[Cash + ST Financial Investments + Accounts Receivable+ Inventory] / ST Liabilities
Liquidity	[Cash + ST Financial Investments + Accounts Receivable] / ST Liabilities
Interest cover	Operating Profit / Financial Expenses
Ratios per employee	
Profit per employee	Earnings before Taxes/Number of Employees
Income per employee	Operating Incomes /Number of Employees
Personnel costs por employee	Personnel Expenses / Number of Employees
Equity per employee	Equity / Number of Employees
Working Capital per employee	Working Capital / Number of Employees
Total Assets per employee	Total Assets / Number of Employees

*Abbreviations: EHNDP (Equity holders by not demanded payments); ST (Short Term); LT (Long Term); C&E (contingencies ans Expenses)

Table 3b. Ratios: Mean and standard deviation for each year

RATIO	FAILED						HEALTHY					
	mean	S.D.	mean	S.D.	mean	S.D.	mean	S.D.	mean	S.D.	mean	S.D.
Activity Ratios.	t	t	t-1	t-1	t-2	t-2	t	t	t-1	t-1	t-2	t-2
Sales growth (%)	-3.2	49.6	8.6	30.7	39.3	110.5	14.5	48.4	29.0	90.0	36.9	104.8
Asset turnover	1.9	1.2	1.7	0.8	1.9	1.2	1.7	1.5	1.6	1.4	1.5	1.2
Productivity	0.8	0.9	1.3	0.6	1.4	0.6	1.9	2.4	1.9	2.2	1.8	1.4
Personnel expenditures (%)	34.8	45.1	25.7	17.2	25.1	17.5	26.9	25.2	25.5	20.3	25.4	20.0

Value Added Growth (%)	-12.0	37.9	11.2	28.5	33.4	91.2	38.9	121.0	51.2	140.3	40.4	110.8
Operating Margin (%)	-11.6	20.4	-1.4	7.1	0.0	5.3	-1.8	79.4	8.2	48.4	8.4	54.6
Net Asset Turnover	9.9	42.0	8.3	11.2	8.7	21.7	4.8	9.7	5.9	37.6	5.9	10.1
Return Ratios.												
ROCE	-4.7	199.2	17.3	95.4	32.7	91.5	21.9	37.2	19.5	36.5	20.5	34.8
ROA	-25.8	59.7	-1.7	10.9	1.0	6.3	2.8	15.5	3.1	11.4	3.7	7.8
ROA before taxes	-24.7	57.0	-1.2	10.8	1.3	7.9	4.6	15.6	4.1	16.2	4.9	10.7
ROE	15.4	167.9	15.2	134.6	12.6	57.4	12.9	85.2	11.5	56.2	5.7	65.2
ROE before taxes	7.8	182.3	8.5	87.2	14.1	70.0	19.2	93.2	19.7	70.0	14.6	44.4
Financial costs (%)	4.8	12.2	2.9	2.2	2.7	2.4	15.0	88.3	13.0	56.1	9.2	53.1
Equilibrium Ratios.												
Working Capital (Mil)	-163	1418	-76	1194	58	498	2049	15018	2669	11733	3159	17870
Need of Working capital (Mil)	-19	1447	206	1154	252	1066	-182	14217	1078	7024	186	13888
Cash (Mil)	-144	912	-282	1258	-194	1004	2231	21583	1591	13070	2973	29669
Equilibrium	-2	27	3	5	3	9	11	72	7	39	5	26
Cinetic Equilibrium Ratios												
Working Capital (days)	-49	243	-9	172	-6	164	361	5608	1139	6770	525	5793
Need of Working capital (days)	-61	266	-14	173	-13	173	-130	1659	298	3437	-65	888
Cash (days)	12	49	6	42	7	45	491	5286	841	6115	590	5993
Clients' Credits (days)	179	753	86	61	89	65	281	1002	215	769	149	234
Clients' Credits due to Sales (days)	178	753	86	61	88	65	234	800	109	128	127	191
Solvency Ratios.												
Debts (%)	107.0	76.5	83.8	22.0	81.4	23.0	68.8	45.4	68.0	43.1	67.9	39.0
Solvency Ratio (%)	-7.0	76.5	16.2	22.0	18.6	23.0	31.2	45.4	32.0	43.1	32.1	39.0
Equity over Permanent funds (%)	64.5	61.0	57.5	48.6	65.5	31.3	75.8	38.4	74.6	34.1	75.7	31.8
Repayment capabilities	2.1	11.0	0.6	0.6	0.6	0.6	5.7	46.5	2.2	10.7	1.8	8.1
Liquidity Ratios.												
Immediate Liquidity	1.6	11.8	0.2	0.9	0.2	0.8	5.5	54.7	4.7	43.6	0.7	2.1
Current Liquidity	8.2	58.4	1.3	1.0	1.3	0.9	7.9	56.1	6.1	43.8	2.2	3.2
Liquidity	2.6	15.7	0.8	0.9	0.8	0.9	7.4	56.1	5.5	43.8	1.5	2.5
Interest cover	-24.6	170.1	-18.5	170.7	-5.9	75.7	30.7	518.6	173.0	1325.0	207.9	1347.2
Ratios per employee.												
Profit per employee (Mil)	-21	60	-4	32	0	13	76	694	34	173	26	146
Income per employee (Mil)	183	234	196	290	199	303	471	1854	302	639	247	457
Personnel expenditures per employee (Mil)	35	81	33	89	44	136	32	51	27	16	25	14
Equity per employee (Mil)	3	49	19	48	22	68	459	2060	390	1962	318	1812
Working Capital per employee (Mil)	79	139	78	130	76	137	255	1001	167	436	136	323
Total Assets per employee (Mil)	140	220	133	214	124	209	930	3928	597	2097	499	1920

	mean	S.D.	mean	S.D.	mean	S.D.	mean	S.D.	mean	S.D.	mean	S.D.
Ratio Trends (%).	t_t-1	t_t-1	t-1_t-2	t-1_t-2	t_t-2	t_t-2	t_t-1	t_t-1	t-1_t-2	t-1_t-2	t_t-2	t_t-2
Activity Ratios Trend.												
Operating Margin (%)	-10.2	18.8	-1.4	6.4	-11.6	20.7	-13	96.4	-0.2	26.4	-13.2	98.1
Equilibrium Ratio Trend.												
Working capital (Mil)	-87	377	-134	1076	-221	1306	-620	7173	-491	8554	-1110	9858
Need for Working capital (Mil)	-224	960	-46	920	-271	1733	-1259	9573	891	9911	-368	7909
Cash (Mil)	137	774	-88	460	49	538	640	10116	-1382	17489	-743	9376
Solvency Ratio Trend.												
Debts (%)	23.3	67.6	2.3	12.9	25.6	71.5	0.9	16.6	0.1	12.8	1.0	19.4
Solvency Ratio (%).	-23.3	67.6	-2.3	12.9	-25.6	71.5	-0.9	16.6	-0.1	12.8	-1.0	19.4
Equity over Permanent funds (%)	7.0	63.2	-8.1	39.6	1.1	68.3	1.2	32.6	-1.1	23.7	0.1	38.0
Repayment capabilities	1.4	10.9	0.0	0.3	1.5	11.0	3.5	44.3	0.3	11.2	3.8	45.8
Liquidity Ratio Trend.												
Immediate Liquidity	1.4	11.8	0.0	0.2	1.4	11.8	0.9	11.3	3.9	41.8	4.8	52.9
Current Liquidity	7.0	58.5	0.0	0.4	6.9	58.5	1.9	16.2	3.9	42.1	5.8	54.5
Liquidity	1.8	15.8	0.0	0.2	1.8	15.8	1.9	16.1	4.0	42.0	5.9	54.4

Table 4.- Results from computational tests

m	P	Constructive deterministic	Greedy-random	GRASP
40	4	0.67346939	0.70408163	0.7755102
	5	0.69387755	0.69387755	0.7755102
	6	0.68367347	0.68367347	0.7755102
	7	0.68367347	0.68367347	0.79591837
	8	0.71428571	0.71428571	0.80612245
65	6	0.67346939	0.71428571	0.80612245
	7	0.69387755	0.69387755	0.81632653
	8	0.70408163	0.70408163	0.82653061
	9	0.70408163	0.70408163	0.84693878
	10	0.73469388	0.68367347	0.85714286

m	P	Constructive deterministic	Greedy-random	GRASP
90	8	0.65306122	0.75510204	0.85714286
	9	0.68367347	0.75510204	0.85714286
	10	0.69387755	0.74489796	0.86734694
	11	0.69387755	0.70408163	0.86734694
	12	0.67346939	0.75510204	0.86734694
105	10	0.64285714	0.74489796	0.87755102
	11	0.60204082	0.75510204	0.87755102
	12	0.60204082	0.71428571	0.87755102
	13	0.60204082	0.7244898	0.8877551
	14	0.59183673	0.69387755	0.87755102
120	12	0.66326531	0.78571429	0.90816327
	13	0.65306122	0.78571429	0.8877551
	14	0.68367347	0.7244898	0.90816327
	15	0.70408163	0.73469388	0.8877551
	16	0.68367347	0.7244898	0.8877551

Table 5 –Value of the objective function f for the different values of p (number of ratios preselected)

Number of preselected ratios (%).	Constructive Deterministic method	Greedy-random method	GRASP method
10	0.66938776	0.71020408	0.80612245
11	0.67959184	0.70408163	0.80204082
12	0.67959184	0.7	0.80612245
13	0.68775510	0.71020408	0.81632653
14	0.67551020	0.71428571	0.82040816
15	0.66530612	0.70816327	0.81224490

Table 6 Number of times each financial ratio is selected by the different algorithms

TOTAL	Deterministic constructive	Random constructive	GRASP	Selected Ratios	
31	12	12	7	Value Added Growth	A
1	1	0	0	Sales growth	A
26	12	12	2	Productivity	A
15	6	6	3	Personnel expenditures (%)	A
4	0	1	3	Operating Margin (%)	A
7	2	2	3	Asset turnover	A
10	3	3	4	Net Asset Turnover	A
3	0	0	3	ROA	R
23	12	6	5	ROA before taxes	R
1	0	1	0	ROE	R
6	0	3	3	ROE before taxes	R
10	2	3	5	ROCE	R
31	9	7	15	Solvency ratio	S
22	12	7	3	Equity over Permanent Funds	S
17	4	6	7	Debt ratio	S
4	0	0	4	Equilibrium	E
1	0	0	1	Working capital (€)	E
2	0	0	2	Need of working capital (€)	E
2	0	2	0	Clients' Credits due to Sales (days)	E_C
3	0	2	1	Income per employee	PE
3	0	2	1	Personnel expenditures per employee	PE
2	0	0	2	Immediate Liquidity	L
1	0	0	1	Cash	E
	Deterministic constructive	Random constructive	GRASP	No Selected Ratios	
	0	0	0	Financial costs %	R
	0	0	0	Working capital (days)	E_C
	0	0	0	Need of Working capital (days)	E_C
	0	0	0	Cash (days)	E_C
	0	0	0	Clients' credit (days)	E_C
	0	0	0	Repayment capability	S
	0	0	0	Current liquidity	L
	0	0	0	Liquidity	L
	0	0	0	Interest cover	L
	0	0	0	Profit per employee	PE
	0	0	0	Equity per employee	PE
	0	0	0	Working capital per employee	PE
	0	0	0	Total Assets per employee	PE
225	75	75	75		

Table 7 *Preselected variables using GRASP*

ROA before taxes_t
ROA_t
Equity over Permanent funds _t-1
Solvency ratio _t-1
Value Added Growth _t-1
Equilibrium_t-1
Debts_t-1_vs_t-2
Working capital_t_vs_t-1
Need for working capital_t_vs_t-1
Debts_t_vs_t-1
Net Asset Turnover_t-1
Solvency ratio_t_vs_t-1
ROCE_t-2
Operating ratio_t

NOTAS

NOTAS

NOTAS
