

IMPACT OF SUPPLY CHAIN CONTRACTS ON INCENTIVES FOR  
LEAD TIME REDUCTION

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**Abstract**

We show that in supply chains where retailer effort can substantially affect sales, longer lead times can result in higher sales for the manufacturer. Hence, manufacturers might not want to reduce lead times even if it was free or inexpensive to do so. Using a one-period model where retailer effort affects sales and is exerted after stocking quantities are determined, manufacturers and retailers have a price-only contract, lead time from the manufacturer to retailer could be reduced to zero at no additional cost (*i.e.* there were no capacity constraints, and there was no added per unit cost of producing with short lead times), and there is no competition (*i.e.*, the retailer and manufacturer have exclusive contracts), we still find conditions under which manufacturers are better off sticking to longer lead times. Our paper highlights how supply chain contracts could act as a potential barrier to reducing lead times.

**Keywords**

Newsvendor, contracts, incentives, lead time, retailer effort



## 1. Introduction

We show that in supply chains where retailer effort can substantially affect sales, the retailer exerts effort after choosing stocking quantity and observing demand, and where the manufacturer and retailer have a wholesale price only contract, longer lead times can result in higher sales for the manufacturer. We also show that, in the presence of retailer effort that is exerted after demand is observed, even if lead time from the manufacturer to retailer could be reduced to zero at no additional cost and there was no competition, there would still be conditions under which manufacturers would be better off sticking to longer lead times. This happens despite the fact that, under our assumptions, because there is no explicit cost of lead time reduction, an integrated company would always reduce lead times). Moreover, when demand is uniformly distributed, the wholesale price in a price-only contract is exogenously determined, and understocking costs equal overstocking costs, the manufacturer would *never* prefer to reduce her<sup>1</sup> lead times.

We compare supply chain performance under two scenarios: when the retailer has to choose stocking quantity before, and after observing demand. We call the former “long lead time case”, and the latter, “short lead time case”. In the long lead time case, the retailer may find it optimal to buy more quantity than under short lead times, knowing that, once he has bought this inventory, he will be likely to work hard to sell it. Thus, the retailer may buy inventory to “induce himself” to exert high effort. Consequently, we find that the effort effect may result in the manufacturer experiencing higher sales in the long lead time case, even if we allow the manufacturer to set her own transfer prices from manufacturer to retailer.

Having identified the phenomenon, we then proceed to discuss different contract forms that could potentially induce manufacturers to reduce their lead times. We show that price plus lump-sum money transfer contracts (i.e. a “franchise fee” plus a constant price contract) would achieve first-best effort and quantity and induce the manufacturer to reduce her lead time to the retailer, and take-or-pay contracts (i.e. a contract that forces the retailer to buy a pre-determined minimum quantity) would induce the manufacturer to reduce lead time but will not achieve first-best stocking quantity or effort.

Our paper is significant managerially because it highlights how supply chain contracts could act as a potential barrier to reducing lead times from manufacturers to distributors and offers alternate contract forms that could be used to overcome this barrier. Consider the automotive industry where each retailer (*i.e.*, car dealership) typically carries products from a single manufacturer (*e.g.*, Ford, Toyota, Buick, etc.) and most industry executives believe that retailer effort (either salesperson effort or local advertising by the car dealer) substantially affects demand at a particular location. In the US, a 40 day lead time for custom cars (or made to order cars) is considered fast, Holweg and Pil (2004). Fisher (1997) argues that, owing to this lead time, more than 90% of customers buy what is in the dealers’ lots; he further argues that the lengthy

lead time has made it difficult for the automobile industry to provide the variety desired by consumers.

At least one automotive manufacturer (Toyota) has developed the ability to produce a car in response to a consumer order within five days, both in the US and Japan, and its total production plus delivery cycle could amount to 14 days, Holweg and Pil (2004). How might this affect the automotive supply chain in the US? On the one hand, some, e.g., Bodestab (1999), have argued that Toyota's ability to shorten lead time for custom car orders could confer advantages that were similar to those that Dell obtained in the PC industry unless competitors responded quickly with similar programs. However, the research reported in this paper argues that the decision might not be that straightforward because, in offering lower lead times to its retailers, Toyota could induce them to carry less inventory, consequently reducing their incentives to apply effort to "push" its product and experiencing lower sales as a consequence. In any case, despite Toyota's theoretical production and delivery capabilities being similar in the US and Japan, its actual delivery times differ significantly by 2000: that year, Toyota North America's average lead time was 77 days, while Toyota Japan was achieving 23 days and had the objective of reducing it to 14 days, Holweg and Pil (2004). On the other hand, if there are exogenous constraints on the amount of inventory that retailers could carry (e.g. wealth or space constraints), then the retailers would not order enough under long lead times for the effort effect to dominate, thus making short lead times the best option. This seems to be consistent with evidence that Toyota's market penetration is much higher in Japan and western Europe than it is in the US, despite Toyota's potential capability to deliver short lead times in all three markets, given that, among a number of other differences with the US, dealerships in Japan and western Europe do seem to be smaller and more limited in their capacity to carry inventory. In fact, during 1999-2000, 48% of cars sold in Europe were built to order and 38% were sold from dealer's lots and 60% of cars in Japan were built to order and 34% were sold from dealer's lots, but only 6% of US cars sold were built to order, while 89% of US cars were sold from dealer's lots, Holweg and Pil (2004). Of course, our intention in this paper is not to offer a final position on whether reducing lead time on custom cars is appropriate, but rather emphasizing that, before lowering lead times, manufacturers should take into account the likely impact on retailers' effort levels to push product and consequently, their sales.

The rest of the paper is organized as follows: In Section 2, we present a literature review. In Section 3, we define the timing, demand structure, cost and revenue parameters. In Section 4 we analyze the case of exogenous wholesale price. In Section 4.1, we formulate the retailer's problem for short lead time. In Section 4.2, we formulate the retailer's problem for long lead time. In Section 4.3, we discuss what drives differences in order quantities between short and long lead time cases. In Section 4.3.1, we define a "safety stock effect" and 4.3.2, an "effort effect". In Section 5, we discuss three different contracts: in Section 5.1 we analyze price plus lump sum money transfer contracts; in Section 5.2, take or pay contracts; in Section 5.3, price only contracts where the manufacturer can propose the price endogenously. Section 6 concludes the paper.

## 2. Literature Review

This paper focuses on a manufacturer and a retailer that deal exclusively (*i.e.* we do not explicitly model a retailer that carries competing products), and a retailer that can exert effort to increase demand after knowing what demand is going to be, and asks the following questions: In this scenario, would there be cases when manufacturers would prefer long to short lead times, even if there was no (explicit) cost of reducing such lead times? What is the effect that retailer effort has on the manufacturer's incentives to reduce lead times?

Others have argued that certain conditions may limit the desirability to manufacturers of reducing lead times. The most frequently cited arguments revolve around costs: *e.g.*, Donohue (2000), assumes that producing with short lead times implies a higher per unit cost to the manufacturer; Fisher and Raman (1996) include a capacity constraint that limits production after information about demand is improved, and assume that increasing that capacity has a cost. In our model, however, reducing lead times is free (*i.e.* there are no capacity constraints, and there is no added per unit cost of producing with short lead times).

Another line of research that deals with the effect competition may have on manufacturer's incentives to reduce lead times. Krishnan, Kapuscinski and Butz (2003) were the first to show analytically that manufacturers might prefer longer lead times when the retailer carries substitute products from other manufacturers who sell with long lead times. The main difference between this paper and Krishnan, Kapuscinski and Butz (2003) is that, in their paper, the existence of substitute, competing products forms the essence of the argument. In fact, retailer effort as modeled in Krishnan, Kapuscinski and Butz (2003) does not increase demand but rather switches customers between products, and thus, absent competing products, the retailer would exert zero effort. Although our conclusions do not disagree with Krishnan, Kapuscinski and Butz (2003), we study a different phenomenon. Both papers are essentially about manufacturers "competing" for the retailer's attention. In our paper, the manufacturer competes for retailer effort with any other activity that the retailer may choose to do or product that the retailer may want to push, since our cost of effort includes the retailer's opportunity cost of exerting effort in other products or chores. In Krishnan, Kapuscinski and Butz (2003), on the other hand, the competition requires two substitute products (*i.e.* products that *consumers* would find approximately equivalent). Since the effects defined in this paper would not only exist in the presence of consumer driven competition, but also apply in more general cases (*e.g.*, a retailer that sells two completely independent products), while in Krishnan, Kapuscinski and Butz (2003) there would be no effort absent competition, our paper represents a generalization of some of the ideas in Krishnan, Kapuscinski and Butz (2003).

In a model with risk neutral parties, no retailer efforts, no competition or substitute products, zero cost of shortening lead time and no capacity constraints, Iyer and Bergen (1997) also find

that manufacturers may still prefer not to implement a short lead time technology. Although we do incorporate the insights from Iyer and Bergen (1997) (calling what drives their result the “safety stock effect”), the main difference with our paper is that Iyer and Bergen (1997) do not model the impact of retailer effort on demand<sup>ii</sup>. Thus, we complement Iyer and Bergen’s ideas, with a new effort effect that interacts with the safety stock effect. We find that the effort effect can reinforce the retailer’s incentive to generate demand, and cause manufacturers to choose long lead times, even when a pure safety stock effect would not suffice. Cachon (2004) also describes the incentives retailers have to push what’s in stock, but his paper’s central concern is a comparison of two types of contracts: a “Push” (*i.e.* the manufacturer is selling to a newsvendor) and a “Pull” (*i.e.* the retailer is buying from a newsvendor) contract. Manufacturing lead times are not an endogenous decision in the models of Cachon (2004) and are held constant.

There are a number of other supply chains contract theory papers that consider retailer’s effort, *e.g.*, Cachon and Lariviere (2002) and Taylor (2000). A comprehensive survey on contract theory applied to supply chains is Cachon (2002). However, other than Krishnan, Kapuscinski and Butz (2003) and (2004), we are not aware of other papers that allow the retailer to exert effort *after* demand has been realized and observed by him, or that consider retailer effort and its impact on the decision to reduce lead times. The timing of effort is key to this paper: the difference in effort effect can only happen if effort is exerted to push sales of what is in stock when demand is low<sup>iii</sup>. In contrast to Krishnan, Kapuscinski and Butz (2003), discussed earlier, Krishnan, Kapuscinski and Butz (2004), is not about reducing lead time: in their model, lead time is constant and exogenous, and the paper’s main concern is about finding coordinating contracts (*i.e.* achieving first best), in the presence of retailer effort and long lead times, while our paper insights are about the implementation of lead time reduction technologies and not necessarily about achieving first best performance.

### 3. The Model

Let there be two risk neutral firms in a supply chain: a manufacturer who sells a product to a retailer that does not carry competing products. The product is sold by the retailer during the period of interest at an exogenously determined price  $r$ . The manufacturer produces the product at a cost of  $c$ , and sells it to the retailer at an exogenous, constant wholesale price  $w$ .

Considering  $w$  to be exogenous and not contingent on lead times may seem artificial, in fact Section 5 explores other contractual arrangements, including the case of allowing the manufacturer to set wholesale prices. However, the exogenous  $w$  assumption has its own rationale: it allows us to focus our analysis in the effect of lead time reductions alone, we wanted to compare long lead time to short lead time production *ceteris paribus* (*i.e.* without changing supply chain contracts concurrently). Moreover, we found case evidence, see Hammond (1994) as an example, that in some cases lead time reduction efforts by manufactures were not

accompanied by changes in wholesale prices. This may partly be due to the difficulty that manufactures or retailers have in quantifying the savings stemming from lead time reductions, and to the fact that, at the time of lead time reduction decisions, wholesale prices are already set and changing them may involve complicating negotiations. Not surprisingly, several other papers in the literature have assumed exogenous wholesale prices (e.g., Van Mieghem (1999), and Krishnan, Kapuscinski and Butz (2003)).

All units left at the end of the period are sold at a marked down price  $s$ . Each time end demand is unmet, the retailer incurs a goodwill loss of  $g$  dollars. This is, in essence, is a “single period” inventory model (although we do allow for effort decisions to be made subsequent to the random component of demand being realized). Finally, the following conditions are met (i)  $w \geq c$ , (ii)  $w > s$ , (iii)  $r + g > w$ .

Demand for the product is denoted by  $\mu e + D$ , where  $D$  is a random variable with a strictly increasing and continuous cumulative distribution, expected value  $E[D]$  and standard deviation  $\sigma$ ;  $e$  is the amount of effort that the retailer expends to increase demand for the product;  $\mu > 0$  is the sensitivity of demand to such effort. Note that we have separated demand in two components: a deterministic component that depends on  $e$ , and a random component that is independent of it. In all cases, we will assume that the retailer can exert effort *after* observing the realization of  $D$ , as in Krishnan, Kapuscinski and Butz (2003) and (2004), and, consequently, the retailer can react to high or low realizations of  $D$ . We also assume that this effort is uncontractible. The effort has an opportunity cost,  $e^2$  (i.e. the square of the effort<sup>iv</sup>) that captures the notion that the marginal cost of effort is increasing in  $e$ . To simplify the analysis, we assume that the cost of effort is identical regardless of lead time or demand realization, a difference with Krishnan, Kapuscinski and Butz (2004)<sup>v</sup>. There are two decision points,  $t = 0$ , and  $t = 1$ . At  $t = 0$ , the distribution of  $D$  and all its parameters are known to both parties, but the actual realization of demand is unknown. At  $t = 1$ , demand is realized, and  $D$  becomes a known number<sup>vi</sup>. Let the retailer’s order be for  $Q$  units. We model lead time reduction by considering two extreme situations. Under long lead time, “L”, the retailer must order  $Q_L$  at  $t = 0$  (i.e. before the realization of demand), and no further replenishment during the selling period is allowed. Under short lead time, “S”, the retailer can order  $Q_S$  at  $t = 1$  –i.e. after he knows the exact demand he is facing-. We assume that there is no extra cost to the manufacturer to offer short lead time, and there are no capacity constraints on either party.

## 4. General Formulation for Exogenous $w$

### 4.1 Short Lead Time

*Retailer’s* *problem*

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At  $t = 1$  -after  $D$  is revealed- the retailer orders  $Q_S^* = D + \mu e_S^*$  (1)

$$\text{where } e_s^* \in \pi_s^{ret} = \underset{er}{\text{ArgMax}} (D + \mu e)(r - w) - e^2 \quad (2)$$

$\pi_s^{ret}$  is the retailer's profit: the first term is quantity sold times margin, the second term is the cost of effort. Solving equation (2) yields:

$$e_s^* = \frac{\mu(r - w)}{2} \quad (3)$$

Equation (3) can be interpreted as coming from "marginal revenue equal marginal cost".

Since at  $t = 0$  we do not know yet the realization of demand,  $Q_s^*$  is a random variable, and

$$E[Q_s^*] = E[D] + \frac{\mu^2(r - w)}{2} \quad (4)$$

### **Proposition I**

$$\begin{array}{lll} \text{(a)} \quad \frac{\partial E[Q_s^*]}{\partial \sigma} = 0 & \text{(b)} \quad \frac{\partial E[Q_s^*]}{\partial w} < 0 & \text{(c)} \quad \frac{\partial E[Q_s^*]}{\partial r} > 0 \\ \text{(d)} \quad \frac{\partial E[Q_s^*]}{\partial \mu} > 0 & \text{(e)} \quad \frac{\partial E[Q_s^*]}{\partial g} = 0 & \text{(f)} \quad \frac{\partial E[Q_s^*]}{\partial s} = 0 \end{array}$$

Throughout the paper, proofs to all propositions are in the appendix.

Proposition I (a), (e), and (f) state that the retailer would not order, on average, any more or less if the standard deviation of demand,  $\sigma$ , the goodwill loss on a lost sale,  $g$ , or the salvage value  $s$  changed. The reasons for this are that: (i) the retailer places orders when residual uncertainty is zero, the manufacturer absorbs all variations on demand; (ii) there are no lost sales of excess inventories. Proposition I (b) states that, the higher the wholesale price,  $w$ , the lower the retailer's order. Proposition I (c) and (d) state that, the higher the retail price,  $r$ , or the sensitivity of demand to retailer's effort,  $\mu$ , the larger the average retailer's order.



## 4.2 Long Lead Time

### Retailer's problem

For long lead time, we will use a “backwards induction” dynamic programming technique. First, we will study what the retailer's effort will be at  $t = I$  under all possible demand realizations, and then we will move to  $t = 0$  to determine the  $Q$  that maximizes his expected profits.

At  $t = I$ , there are two cases:

I) If  $D > Q$ , the retailer solves<sup>vii</sup>

$$\text{Max}_e Y_I(Q) = Q(r - w) - (D + \mu e - Q)(g) - (e)^2 \quad (5)$$

$Y_I(Q)$  is the retailer's profit function. The first term is the quantity sold times its margin, the second, lost sales times goodwill loss incurred per unit of sale lost, the third, the cost of effort. Equation (5), by, inspection, yields  $e^* = 0$ . In words, if there is a stockout, the retailer would not exert any effort since this would just result in more lost goodwill, a cost of effort, and no additional sales (because  $D > Q$  even if  $e = 0$ )

II) If  $D \leq Q$ , the retailer solves

$$\text{Max}_e Y_{II}(Q) = (D + \mu e)(r - w) + (Q - (D + \mu e))(s - w) - (e)^2 \quad (6)$$

s.t.

$$D + \mu e \leq Q \quad (7)$$

Again,  $Y_{II}(Q)$  is the retailer's profit function. The first term is net sales; the second term is net loss generated by quantity bought for  $w$  and salvaged at  $s$ ; the third term is the cost of effort. The constraint (7) states that sales cannot be more than  $Q$ . The solution to (6) is

$$e^* = \min\left\{\frac{\mu(r-s)}{2}, \frac{(Q-D)}{\mu}\right\}, \quad (8)$$

Now, going back to  $t = 0$ , the optimal order quantity solves:

$$E[\pi_L^{ret}] = \text{Max}_Q Z \quad (9)$$

$$\text{Where } Z = \left\{ \begin{array}{l} Z_I \text{ if } Q \leq \lambda, \text{ and } Q \geq \frac{\mu^2(r-s)}{2} \\ Z_{II} \text{ if } Q \leq \lambda, \text{ and } Q < \frac{\mu^2(r-s)}{2} \\ Z_{III} \text{ if } Q > \lambda, Q - \frac{\mu^2(r-s)}{2} < \lambda, \text{ and } Q \geq \frac{\mu^2(r-s)}{2} \\ Z_{IV} \text{ if } Q > \lambda, Q - \frac{\mu^2(r-s)}{2} < \lambda, \text{ and } Q < \frac{\mu^2(r-s)}{2} \\ Z_V \text{ if } Q > \lambda, \text{ and } Q - \frac{\mu^2(r-s)}{2} \geq \lambda \end{array} \right\} \quad (10)$$

and  $\lambda$  is the upper bound of the demand distribution ( $\lambda$  need not be finite). Note that (i)  $Z_{III}, Z_{IV}, Z_V$  are only feasible if  $\lambda < \infty$ , and (ii)  $Z_{IV}$  can only be feasible if  $\frac{\mu^2(r-s)}{2} > \lambda$ .

$Z$  within each region is defined as:

$$\begin{aligned} Z_I = & \int_0^{\frac{\mu^2(r-s)}{2}} [Y_{IIa}(Q_I)] f(D) dD + \int_{\frac{\mu^2(r-s)}{2}}^{Q_I} [Y_{IIb}(Q_I)] f(D) dD \\ & + \int_{Q_I}^{\lambda} [Y_{IIIa}(Q_I)] f(D) dD \end{aligned} \quad (11)$$

$$Z_{II} = \int_0^{Q_{II}} [Y_{IIb}(Q_{II})] f(D) dD + \int_{Q_{II}}^{\lambda} [Y_I(Q_{II})] f(D) dD \quad (12)$$

$$Z_{III} = \int_0^{Q_{III} - \frac{\mu^2(r-s)}{2}} [Y_{IIIa}(Q_{III})] f(D) dD + \int_{Q_{III} - \frac{\mu^2(r-s)}{2}}^{\lambda} [Y_{IIIb}(Q_{III})] f(D) dD \quad (13)$$

$$Z_{IV} = \int_0^{\lambda} [Y_{IVb}(Q_{IV})] f(D) dD \quad (14)$$

$$Z_V = \int_0^{\lambda} [Y_{Va}(Q_V)] f(D) dD \quad (15)$$

Equation (12) states that the retailer chooses  $Q$  to maximize  $Z$ , its profit function. Equation (14) is profit function in region I: the first term considers when the case stated in equation (9), that is, when  $D + \frac{\mu^2(r-s)}{2} \leq Q$  and the constraint on  $Q$  does not bind at  $t = I$ ; the second term corresponds to the case stated in equation (10), that is, when  $D + \frac{\mu^2(r-s)}{2} > Q$  and the constraint on  $Q$  is binding at  $t = I$ ; the third term corresponds to the case stated in equation (5), that is, when  $D > Q$  and there is a stockout. A similar logic applies to regions II, III, IV, and V. Notice how, depending on the parameter values, certain cases may be infeasible (*e.g.*, the constraint on  $Q$  may always bind at  $t = I$ , etc.). As mentioned in the literature review, the model here presented is similar in spirit to that in Krishnan, Kapuscinski and Butz (2004).

In the appendix we show that, if  $\mu = 0$ , the problem reduces to the regular newsvendor, which is intuitive (once effort does not matter, the retailer facing a long lead time is a pure newsvendor).

**Proposition II**

(a) If the demand distribution has an infinite upper bound, the retailer's profit function under long lead times is continuously differentiable and concave in  $Q$ .

(b) If the demand distribution has a finite upper bound, then the retailer's profit function under long lead times is:

- (i) continuously differentiable in  $Q$
- (ii) concave in  $Q$  in regions I,...IV
- (iii) linear in  $Q$ , with negative slope, in region V.

To understand the difference between Proposition II (a) and (b), note that if the demand distribution has an infinite upper bound, then regions III, IV, and V are feasible. Proposition II (iii) implies that region V could never be optimal: since the retailer's profits are decreasing in  $Q$  everywhere in this region, the retailer would simply decrease  $Q$  until either region III or IV are the proper objective functions. Proposition II (i) and (ii) implies that the optimum must be in either of regions I..IV, is unique, and at the point where the derivative of the objective function equals zero (that is, the derivative that corresponds to a valid region, depending on the parameter values).

**Proposition III**

$$(a) \text{ both } \frac{\partial Q_L^*}{\partial \sigma} > 0 \text{ or } \frac{\partial Q_L^*}{\partial \sigma} \leq 0 \text{ can happen} \quad (b) \frac{\partial Q_L^*}{\partial w} < 0 \quad (c) \frac{\partial Q_L^*}{\partial r} > 0$$

$$(d) \text{ both } \frac{\partial Q_L^*}{\partial \mu} > 0 \text{ or } \frac{\partial Q_L^*}{\partial \mu} \leq 0 \text{ can happen} \quad (e) \frac{\partial Q_L^*}{\partial g} \geq 0 \quad (f) \frac{\partial Q_L^*}{\partial s} \geq 0$$

Notice how, Proposition I (a) shows that  $\frac{\partial E[Q_s^*]}{\partial \sigma} = 0$  while Proposition III (a) states that  $\frac{\partial Q_L^*}{\partial \sigma}$  might not necessarily be equal to zero. Depending on the sign of  $\frac{\partial Q_L^*}{\partial \sigma}$ , an increase in  $\sigma$  may either benefit or hurt the manufacturer offering long lead times. In fact, for items with high service levels, an *increase* in  $\sigma$  could lead retailers under long lead time to order *more* (this is formally shown in the appendix for the case of  $\mu = 0$ ). When this holds, manufacturers with high

enough service levels facing increasingly variable demands would prefer offering long lead times, a counter intuitive result, since the supply chain wide benefits of reducing lead times increase with demand variability. Propositions III (b) and (c) show that the sign of these partial derivatives is the same for short and long lead times. Proposition III (d) shows that, under certain circumstances, an increase in  $\mu$  may decrease  $Q_L^*$  (although this was never the case for short lead times).

Proposition III (e) and (f) show how an increase in  $g$  or  $s$  would increase  $Q_L^*$ , either to mitigate lost sales costs, or to take advantage of higher salvage values (remember that, because with zero lead time there are no lost sales or extra stock,  $g$  and  $s$  did not have an effect in  $E[Q_S^*]$ ).

#### **Proposition IV**

- a)  $Var(e_L^*) \geq Var(e_S^*) = 0$
- b)  $Var(Q_L^*) = 0 < Var(Q_S^*) = Var(D)$
- c) For a fixed wholesale price  $w$ ,  $E[\pi_L^{ret}] \leq E[\pi_S^{ret}]$

Propositions IV (a) and (b) would be relevant if the parties were not risk neutral, and show how our model predicts an increase in order variability when manufacturers reduce lead time. Also, Proposition IV (b) shows that manufacturer sales are, for short lead times, random, and, for long lead times, deterministic. Note that the manufacturer's profits are her margins times the quantity sold, or simply:

$$\pi_L^{man} = (w - c) Q_L^* \quad (16)$$

and

$$E[\pi_S^{man}] = (w - c) E[Q_S^*] \quad (17)$$

Since the manufacturer is risk neutral, in this section we have considered  $w$  to be exogenous, and in our model  $c$  does not change with lead time, comparing manufacturer's profits is equivalent to comparing  $E[Q_S^*]$  and  $Q_L^*$ .

Proposition IV(c) means that, under the assumptions of this section (*i.e.* exogenous  $w$ ), retailers would always prefer shorter lead times. In addition, the proof of (c) can also be used to prove that, under the assumptions of this model, short lead times are first best.

### 4.3 What Drives Differences in Order Quantities Between Short and Long Lead Time Cases

Absent retailer effort, Iyer and Bergen (1997) show that for the Normal distribution there exist cases where  $Q_L^* < E[Q_S^*]$  and cases where  $Q_L^* > E[Q_S^*]$ . This is still true once retailer effort is introduced, and general demand distributions are allowed (see the appendix for examples). Therefore, it is possible for manufacturers to lose by moving to short lead times, answering the basic question we proposed in the beginning of the paper. We now proceed to examine the mechanisms driving the result, and the conditions under which long lead times may be preferred.  $Q_L^*$  can exceed  $E[Q_S^*]$  for two reasons. One, which we term “safety stock effect” has been discussed by Iyer and Bergen (1997). The second, which we term “effort effect”, is introduced by us. The safety stock effect refers to a retailer’s willingness to order additional inventory to buffer against demand uncertainty when lead times are long. The effort effect refers to the retailer’s willingness to order additional stock when lead times are long to induce himself to exert additional effort after demand uncertainty has been resolved. To isolate and precisely define these two effects, we proceed as follows: first, we precisely define the safety stock effect; next, we define parameter values such that the safety stock effect would be non-existent, and call  $Q_L^* - E[Q_S^*]$  in these circumstances the “pure” effort effect.

#### 4.3.1. The Pure “Safety Stock Effect”

Let retailer effort be non-existent or be irrelevant to demand (*i.e.*, let  $\mu = 0$ ) -a set up that is a generalization of Iyer and Bergen (1997)’s normal distribution model-. In this case,  $Q_L^*$  is exactly the solution to a common newsvendor problem (see appendix for proof). In fact,  $Q_L^*$  solves:

$$\int_Q^\infty f(D)dD = \frac{(w-s)}{(r+g-s)} \quad (18)$$

With long lead times, the retailer, facing uncertainty about the realization of demand, may buy more or less  $Q$ , depending on the value of the parameters (specifically, depending on the ratio of overage cost to overage plus underage cost).

Let  $\lambda$  be the upper bound of  $f(D)$  (need not be finite). Then  $Q_L^* \in \{0, \lambda\}$ , but we know from previous results that, if  $\mu = 0$ , then  $E[Q_S^*] = E[D]$ . Therefore, even without effort present, it is possible to have  $Q_L^* \geq E(Q_S^*)$  or  $Q_L^* < E(Q_S^*)$ . For example, if this was the only effect present, for normally distributed demand, the manufacturer would only benefit from shortening the lead time of products with critical fractiles below 0.5. This is, in essence, a key result that Iyer and Bergen (1997) present.

We define the “pure safety stock effect” as  $\Delta Q^* = Q_L^* - E[Q_S^*] = Q_L^* - E[D]$  when  $\mu = 0$ , or as the difference in quantities ordered between the long lead time and short lead time cases when retailer effort are irrelevant.

### 4.3.2 The Pure “Effort Effect”

Next, we will define a set of parameter such that the safety stock effect is zero. In the absence of effort, let:

$$\frac{w-s}{g+r-s} = 1 - F(\bar{Q}_L) \quad (19)$$

$$\text{where } F(\bar{Q}_L) = \int_0^{\bar{Q}_L} f(D) dD \quad (20)$$

$$\text{and } \bar{Q}_L = E[D]. \quad (21)$$

Then, we know from the newsvendor that  $Q_L^* = E[D]$ , which would then imply that  $Q_L^* = E(Q_S^*)$ , making, as predicted, the safety stock effect zero.

However, in the presence of effort it is no longer necessarily true that  $Q_L^* = E(Q_S^*)$  under the same cost parameters as above. That is, even if the “critical fractile” is such that a pure newsvendor would order the mean demand,  $Q_L^*$  may be different from  $E[Q_S^*]$ .

We define as a pure “effort effect” as  $\Delta Q_i^* = Q_i^* - E[D]$ ,  $i \in \{L, S\}$ , when the above condition on the ratio of overage and underage cost holds. In words, the pure “effort effect” is the change in  $Q_i^*$  due purely to effort exerted by the retailer, independent of any safety stock effects. Note how the “effort effect” can work for both the long lead time and short lead time cases, while

the “safety stock effect” is defined as the difference between these two cases absent any effort. From the proof of Proposition II (b), one can notice that, under the short lead time scenario, the retailer exerts a constant effort, while under the long lead time scenario, depending on  $Q_L$ , the parameter values, and the demand outcome, the retailer can either exert more or less effort than under short lead time. This is a key element of our analysis: the effort effect does not necessarily act with equal magnitude with short and long lead times.

### **Proposition V**

If  $\mu$  is large enough, and

- a) the upper bound of  $f(D)$  is  $\lambda < \infty$ , then  $Q_L^* = E[Q_S^*]$ : the “safety stock effect” will eventually be 0, and the “effort effect” will eventually be equal for both cases.
- b) the upper bound of  $f(D)$  is  $\lambda = \infty$ , as  $\mu$  becomes larger and larger,  $Q_L^*$  asymptotically approximates  $E[Q_S^*]$ : the “safety stock effect” will be negligible, and the difference in “effort effects” will become smaller and smaller.

Proposition V states that for large enough values of  $\mu$ , the safety stock effects are either equal or very close. This can be understood the following way: if  $\mu$  is large enough, then the retailer is able to easily “generate” demand, thus increasing  $Q$ . At very large values of  $\mu$ ,  $Q$  is so large that the randomness in  $f(D)$  becomes less and less relevant, and the solution approximates the “deterministic effort” short lead time case. However, as an example with the uniform distribution will show, things can be very different for smaller values of  $\mu$ .

### **4.3.3 An example: the case of uniformly distributed demand**

If demand is uniformly distributed between  $[0, \lambda]$ ,  $g > 0$ , and overage cost = underage cost (i.e.  $\frac{w-s}{g+r-s} = \frac{1}{2}$ , thus making the safety stock effect zero), then, if  $\mu \geq \sqrt{\frac{\lambda}{w-c}}$ ,  $Q_L^* = E[Q_S^*]$ , just as Proposition V predicted.

However, if  $0 < \mu < \sqrt{\frac{\lambda}{w-c}}$ , then  $Q_L^* > E[Q_S^*]$ . Therefore, for the uniform distribution, the pure effort effect is *always* larger for long than for short lead times.

In summary, even if overage and underage costs are equal, there will be many cases where the manufacturer will prefer to continue producing with long lead times. This example shows how,



for cases where the retailer's effort can influence demand (but this influence is below a threshold), the effort effect can be significantly reinforced by the retailer's desire to push what he has in stock –which only happens in long lead times-, thus making him buy more from the manufacturer than if he was offered short lead times.

## 5. Analysis of Alternate Contracts

Having detected the existence of a problem we now proceed to discuss potential remedies. In other words, if the manufacturer resists lead time reductions fearing lower profits/sales, how can we change supply chain contracts to overcome this?

### 5.1 Price Plus Lump Sum Money Transfer Contracts

If we allow the principal to endogenously set  $w$  (the transfer price) and a lump-sum money transfer  $T$ , then the manufacturer would always prefer short lead times regardless of the allocation of bargaining power or who is proposing the contract. The solution would simply be to set  $w = c$ , and let the parties split the profits via  $T$ . In this case, because it would be optimal for an integrated firm to reduce lead time (because it leads to higher channel profits), the parties would do so and split the larger benefit accordingly. One way to interpret this is to imagine that the retailer pays the manufacturer a franchise fee, and that, in exchange for this, the manufacturer sells at cost, and with short lead time. This arrangement would truly achieve first best: not only it would make the manufacturer always prefer short lead time, it would also eliminate double marginalization as defined in Spengler (1950), Pasternack (1985).

It is interesting to note that many leaders in short lead time, for example, Norwalk in the furniture business, Salmon, Raman et al (1998), Zara in apparel, Ghemawat and Nueno (2003), Dell in the PC business, Rivkin and Porter (1999), make the majority of their selling either (i) directly to the customers, (ii) through company owned retail shops, or (iii) through franchisees, and have either not succeed in selling to (Norwalk, Dell), or not tried to sell to (Zara), independent retailers.

### 5.2 Take or Pay Contracts

As suggested in Krishnan, Kapuscinski and Butz (2003), a potentially simple way to induce the manufacturer to reduce lead times when she fears losing sales is to force the retailer to agree to buy a minimum amount of product as a condition for the manufacturer to offer short lead times. In our model, just as in Krishnan, Kapuscinski and Butz (2003), if the manufacturer requires the retailer to buy, at least,  $Q_L^*$  in exchange for short lead time, then both parties would be better off than with long lead times, since offering short lead times would allow both parties to

increase quantities in response to high demand. If bargaining power is split, then it would be possible to find some bargained order quantity that would split revenues between manufacturer and retailer to accommodate for this. It is important to notice that this is a second best solution (*i.e.* it does not achieve coordination), since although such contracts would induce the manufacturer to reduce lead times, the minimum order quantity constrains the retailer's problem and thus limits his options (with respect to no minimum order quantity).

### 5.3 Price Only Contracts with Endogenous Constant Pricing

A third idea is simply to allow the manufacturer to set  $w$  endogenously, allowing her to charge a different (potentially higher) price when moving to short lead times. This idea –allowing the manufacturer to propose the retailer price only contracts, with different prices for short and long lead times- seems doable in practice, but is complex analytically: it turns out that the manufacturer's long lead time problem need not be well behaved, not allowing us to reach general conclusions. Nevertheless, we will be able to show by example that these contracts may not solve the problem, making, in some cases, the price setting manufacturer better off staying with long lead times.

#### 5.3.1 Short Lead Time

##### Manufacturer's problem<sup>viii</sup>

$$\pi_s^{man} = \underset{w}{\text{Max}} (w-c)E[Q_s^*(w)] = (w-c)(E[D]) + \frac{(r-w)\mu^2}{2} \quad (22)$$

##### Proposition VI

Under short lead time, the manufacturer's problem is concave in  $w$ .

Using Proposition VI, making  $\frac{\partial \pi_s^{man}}{\partial w} = 0$  and solving yields

$$w^* = \frac{E[D]}{\mu^2} + \frac{(c+r)}{2} \quad (23)$$

Equation (26) shows that the optimum transfer price  $w$  is increasing in the expected value of the distribution, the retail price and the manufacturer's costs and, as expected, decreasing in  $\mu$ , since a larger  $\mu$  makes it more lucrative to reduce  $w$  to induce more retailer effort.

### 5.3.2 Long Lead Time

Although the general formulation of the manufacturer's problem under long lead times looks similar to the short lead time problem, things get much more complicated. Lariviere and Porteus (2001) show that, even without retailer's effort,  $\pi_L^{man}(w)$  need not be unimodal even if  $Q_L^*(w)$  was concave in  $w$  (which, by the way, is not the case for most common distributions including the normal). Nevertheless, absent retailer's effort, Lariviere and Porteus (2001) are able to find an intuitive condition based on the demand distribution's generalized failure rate. In the presence of retailer's effort, however, we are unable to find such a tractable, general condition that would guarantee that  $\pi_L^{man}(w)$  is concave in  $w$ . The fact that, for different values of  $w$ , the shape of the formula for  $Q_L^*(w)$  changes exacerbates the difficulty in finding such a general condition. Next, we proceed to present the formulation of the manufacturer's problem for a general case.

#### Manufacturer's problem

$$E[\pi_L^{man}]^* = \underset{w}{\text{Max}} (w - c)(Q_Z^*(w)) \quad (24)$$

s.t.

$$Q_Z^*(w) \in \underset{Q}{\text{ArgMax}} E[\pi_L^{ret}] = Z \quad (25)$$

Note that, for each  $w$ , one must find  $Q_Z^*(w)$ , by searching  $Z$  in all possible regions of validity. Equation (24) states that the manufacturer will set  $w$  to maximize her profits given that the retailer will respond to each  $w$  by buying her profit maximizing quantity  $Q_Z^*(w)$ , as stated by equation (25).

### 5.3.3 Short vs. Long: a numerical example with Uniformly distributed demand

Although we cannot find a, general, and tractable condition that will guarantee that the problem above is unimodal, we proceed now to present a numerical example where the manufacturer would prefer longer lead times under a wide range of values of  $\mu$ . In other words, our example shows that allowing the manufacturer to set  $w$  does not necessarily make her prefer short lead times.

Let demand be uniformly distributed between  $[0, 1000]$ , and  $r = 5$ ,  $s = 2.5$ ,  $g = 1$ ,

$$\text{I) if } \mu \leq 14.28, \quad (26)$$

$$\text{then } [\pi_L^{man}]^* \leq E[\pi_S^{man}]^*. \quad (27)$$

$$\text{II) If } 14.28 < \mu < 10\sqrt{10} \approx 31.62$$

$$\text{then } [\pi_L^{man}]^* > E[\pi_S^{man}]^*. \quad (28)$$

$$\text{III) If } \mu \geq 10\sqrt{10} \approx 31.62$$

$$\text{then } [\pi_L^{man}]^* = E[\pi_S^{man}]^*. \quad (29)$$

That is, for small  $\mu$ , the manufacturer will achieve higher profits offering short lead times. For intermediate values of  $\mu$ , the manufacturer will achieve higher profits offering long lead times, and for large values of  $\mu$ , she will be indifferent between offering short and long lead times.

To sum up, although it may seem, a priori, that the manufacturer could simply offer short lead times at a higher wholesale price than under long lead times, this may not be optimal for her. The wholesale price is forced to serve three purposes (i) induce retailer's effort (which indirectly affects the retailer's quantity ordered), (ii) induce retailer's quantity ordered directly, and (iii) split the total profits between the manufacturer and retailer. The multiple purposes that wholesale price fulfils can create cases when long lead times are more lucrative for the manufacturer. For example, the "effort inducing" effect of  $w$  can result in higher effort under long lead times. This has an intuitive explanation: under short lead times, the retailer's marginal return from effort is a constant  $\mu(r-w)$ . In the long lead time cases, the retailer's effort depends on the realization of demand: it may either be more than short lead time effort, because the marginal return to effort can be  $\mu(r-s) \geq \mu(r-w)$ , or less than lead time effort, zero if "independent demand" is large enough (see proof of Proposition IV). Under certain scenarios, the manufacturer can effectively exploit the high effort exerted by the retailer when he finds himself with excess inventory making her better off staying with long lead times. For example, in region III, since  $Q > \lambda$ , there are no cases of zero effort, but there exist cases of more effort (than when short lead times are offered). Finally, if  $\mu$  is large enough, it will be optimal for the manufacturer to make the retailer stock a quantity that makes region IV valid, thus achieving the same results under long and short lead

times. This does not contradict intuition: for  $\mu$  large enough, generating “deterministic” demand is easy for the retailer, and therefore he will stock so much that the randomness in demand becomes irrelevant for the retailer’s effort.

## 6. Conclusion

By now, a lot has been said about the cost of long lead times. Reducing lead times could lead to great savings in stockouts and overstocks, allowing products better fitted to customer needs and even mass customization and build-to-order in products ranging from jeans to cars. Movements like Quick Response in the apparel industry and Continuous Replenishment in the grocery industry have been striving to reduce lead times as an essential component of the set of practices they preach. Icons like Dell and Zara have built successful business models where short lead times are a key element of their overall strategy. While we agree about the potential for savings and better products that reduced lead times could achieve, this paper presents both a caveat and a possible explanation for the amazing resilience that long lead times have shown in certain cases, even in industries like automotive, where the potential benefits seem among the largest. Such resilience could be attributed to many causes. Our contribution is to highlight, isolate and precisely define two incentives based potential causes that we think should be considered.

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## Appendix-Proofs

### Proof of Proposition I

Taking the derivatives of  $E[Q_S^*] = E[D] + \frac{\mu^2(r-w)}{2}$  with respect to each parameter yields the result.

### Proof of Proposition 1I

Let  $\lambda$  be the upper bound of the demand distribution,  $D$  ( $\lambda$  need not be finite). The proof considers the objective function under the five possible cases, calculates the derivatives with respect to  $Q$  in each region, and, finally, evaluates these derivatives at the transition points to show continuity:

$$\text{I) } Q_L \leq \lambda \text{ and } Q_L \geq \frac{\mu^2(r-s)}{2}$$

$$\text{II) } Q_L \leq \lambda \text{ and } Q_L < \frac{\mu^2(r-s)}{2}$$

$$\text{III) } Q_L > \lambda, \text{ and } Q_L - \frac{\mu^2(r-s)}{2} < \lambda, \text{ and } Q_L \geq \frac{\mu^2(r-s)}{2} \text{ (only active if } \lambda \text{ is finite)}$$

$$\text{IV) } Q_L > \lambda, \text{ } Q_L - \frac{\mu^2(r-s)}{2} < \lambda, \text{ and } Q_L < \frac{\mu^2(r-s)}{2} \text{ (only active if } \lambda \text{ is finite and } \frac{\mu^2(r-s)}{2} > \lambda)$$

$$\text{V) } Q_L > \lambda, \text{ and } Q_L - \frac{\mu^2(r-s)}{2} \geq \lambda \text{ (only active if } \lambda \text{ is finite)}$$

and proceeds to show that  $\frac{\partial^2 E[\pi_L^{ret}]}{\partial^2 Q} < 0$  in each case.

$$\text{I) } Q_L \leq \lambda \text{ and } Q_L \geq \frac{\mu^2(r-s)}{2}$$



$$\frac{\partial E[\pi_L^{ret}]}{\partial Q} = (s-w) \int_0^{Q-\frac{\mu^2(r-s)}{2}} f(D)dD + (r-w) \int_{Q-\frac{\mu^2(r-s)}{2}}^Q f(D)dD$$

$$+ (r+g-w) \int_Q^\infty f(D)dD + \int_{Q-\frac{\mu^2(r-s)}{2}}^Q \left(\frac{2(D-Q)}{\mu^2}\right) f(D)dD$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial^2 Q} = -gf(Q) - \frac{2}{\mu^2} \int_{Q-\frac{\mu^2(r-s)}{2}}^Q f(D)dD < 0$$

II)  $Q_L \leq \lambda$  and  $Q_L < \frac{\mu^2(r-s)}{2}$

$$\frac{\partial E[\pi_L^{ret}]}{\partial Q} = \int_0^Q \left((r-w) + 2\left(\frac{D-Q}{\mu^2}\right)\right) f(D)dD + (r+g-w) \int_Q^\infty f(D)dD$$

and

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial^2 Q} = \int_0^Q -\frac{2}{\mu^2} f(D)dD - f(D)g < 0$$

III)  $Q_L > \lambda$ , and  $Q_L - \frac{\mu^2(r-s)}{2} < \lambda$ , and  $Q_L \geq \frac{\mu^2(r-s)}{2}$  (only active if  $\lambda$  is finite)

$$\frac{\partial E[\pi_L^{ret}]}{\partial Q} = \int_0^{Q-\frac{\mu^2(r-s)}{2}} (s-w)f(D)dD + \int_{Q-\frac{\mu^2(r-s)}{2}}^\lambda \left((r-w) + 2\left(\frac{D-Q}{\mu^2}\right)\right) f(D)dD$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial^2 Q} = \int_{Q-\frac{\mu^2(r-s)}{2}}^\lambda -\left(\frac{2}{\mu^2}\right) f(D)dD < 0$$

IV)  $Q_L > \lambda$ ,  $Q_L - \frac{\mu^2(r-s)}{2} < \lambda$ , and  $Q_L < \frac{\mu^2(r-s)}{2}$  (only active if  $\lambda$  is finite and  $\frac{\mu^2(r-s)}{2} > \lambda$ )

$$\frac{\partial E[\pi_L^{ret}]}{\partial Q} = \int_0^\lambda ((r-w) + 2\frac{(D-Q)}{\mu^2})f(D)dD$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial^2 Q} = \int_0^\lambda -\frac{2}{\mu^2}f(D)dD = -\frac{2}{\mu^2} < 0$$

V)  $Q_L > \lambda$ , and  $Q_L - \frac{\mu^2(r-s)}{2} \geq \lambda$  (only active if  $\lambda$  is finite)

$$\frac{\partial E[\pi_L^{ret}]}{\partial Q} = (s-w) < 0$$

To finalize the proof, we need to show that the left and right hand side derivatives at points with either  $Q_L = \lambda$ ,  $Q_L = \frac{\mu^2(r-s)}{2}$ , and  $Q_L - \frac{\mu^2(r-s)}{2} = \lambda$  are equal, thus making the objective function continuously differentiable.

a)  $Q_L = \lambda$ , and  $Q_L \geq \frac{\mu^2(r-s)}{2}$ , the boundary between regions I and III.

$$\begin{aligned} \left[ \frac{\partial E[\pi_L^{ret}]}{\partial Q} \right]_{Q=\lambda}^+ &= \left[ \frac{\partial E[\pi_L^{ret}]}{\partial Q} \right]_{Q=\lambda}^- = \\ &= \int_0^{\lambda - \frac{\mu^2(r-s)}{2}} (s-w)f(D)dD + \int_{\lambda - \frac{\mu^2(r-s)}{2}}^\lambda ((r-w) + 2\frac{(D-\lambda)}{\mu^2})f(D)dD \end{aligned}$$

b)  $Q_L = \lambda$ , and  $Q_L < \frac{\mu^2(r-s)}{2}$ , the boundary between regions II and IV.

$$\left[ \frac{\partial E[\pi_L^{ret}]}{\partial Q} \right]_{Q=\lambda}^+ = \left[ \frac{\partial E[\pi_L^{ret}]}{\partial Q} \right]_{Q=\lambda}^- = \int_0^\lambda ((r-w) + 2\frac{(D-\lambda)}{\mu^2})f(D)dD$$

c)  $Q_L = \frac{\mu^2(r-s)}{2}$  and  $Q_L \leq \lambda$ , the boundary between regions I and II.

$$\begin{aligned} \left[ \frac{\partial E[\pi_L^{ret}]}{\partial Q} \right]_{Q=\frac{\mu^2(r-s)}{2}}^+ &= \left[ \frac{\partial E[\pi_L^{ret}]}{\partial Q} \right]_{Q=\frac{\mu^2(r-s)}{2}}^- = (r-w) \int_0^{\frac{\mu^2(r-s)}{2}} f(D) dD \\ &+ (r+g-w) \int_{\frac{\mu^2(r-s)}{2}}^{\infty} f(D) dD + \int_0^{\frac{\mu^2(r-s)}{2}} \left( \frac{2(D - \frac{\mu^2(r-s)}{2})}{\mu^2} \right) f(D) dD \end{aligned}$$

d)  $Q_L = \frac{\mu^2(r-s)}{2}$  and  $Q_L > \lambda$ , the boundary between regions III and IV.

$$\left[ \frac{\partial E[\pi_L^{ret}]}{\partial Q} \right]_{Q=\frac{\mu^2(r-s)}{2}}^+ = \left[ \frac{\partial E[\pi_L^{ret}]}{\partial Q} \right]_{Q=\frac{\mu^2(r-s)}{2}}^- = \int_0^{\lambda} \left( (r-w) + 2 \left( \frac{D - \frac{\mu^2(r-s)}{2}}{\mu^2} \right) \right) f(D) dD$$

e)  $Q_L > \lambda$ , and  $Q_L - \frac{\mu^2(r-s)}{2} = \lambda$ , the boundary between regions III and V.

$$\left[ \frac{\partial E[\pi_L^{ret}]}{\partial Q} \right]_{Q=\frac{\mu^2(r-s)}{2} + \lambda}^+ = \left[ \frac{\partial E[\pi_L^{ret}]}{\partial Q} \right]_{Q=\frac{\mu^2(r-s)}{2} + \lambda}^- = (s-w)$$

### Proof of Proposition III

(a) Proof by example. Let  $\mu = 0$ , and demand be normally distributed with mean  $E[D]$  and standard deviation  $\sigma$ . Latter in this appendix we show that, when  $\mu = 0$ , the retailer's problem reduces to the regular newsvendor.

If we make  $z = \frac{(Q_L - E[D])}{\sigma}$

then  $\Phi(z^*) = \frac{(w-s)}{(r+g-s)}$ , where  $\Phi(z^*)$  is the cdf of the standard normal distribution.

then,  $Q_L^* = z_L^* \sigma + E[D]$

Therefore,  $Q_L^*$  is proportional to the standard deviation of demand.

However, whether  $Q_L^*$  increases or decreases with  $\sigma$  depends on the values of the parameters. If the critical fractile is larger than 0.5, then  $z_L^* > 0$  and  $Q_L^*$  increases with  $\sigma$ . If, on the other hand, the critical fractile is smaller than 0.5,  $Q_L^*$  decreases with  $\sigma$ .

(b), (c), (d), (e), and (f) are proven the following way:

From the implicit function theorem, we know that, if  $x$  is a parameter in the objective function, then:

$$\frac{dQ_L^*}{dx} = - \frac{\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial x}}{\frac{\partial^2 E[\pi_L^{ret}]}{\partial^2 Q_L^*}}$$

But, from Proposition II, we know that in all cases  $\frac{\partial^2 E[\pi_L^{ret}]}{\partial^2 Q^*} < 0$ .

Therefore, the sign of the crosspartial derivative in the numerator is equal to the sign of the total derivative.

Next, we will calculate all the crosspartial derivatives for each of the four possible objective functions where the optimum may happen –i.e. for regions I...IV.

$$I) Q_L^* < \lambda \text{ and } Q_L^* \geq \frac{\mu^2(r-s)}{2}$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial w} = -1 < 0$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial \mu} = - \frac{4}{\mu^3} \int_{Q - \frac{\mu^2(r-s)}{2}}^Q (D-Q)f(D)dD = \frac{2}{\mu} \int_{Q - \frac{\mu^2(r-s)}{2}}^Q -\frac{2}{\mu^2}(D-Q)f(D)dD$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial r} = \int_Q^\infty f(D) dD + \int_{Q - \frac{\mu^2(r-s)}{2}}^Q f(D) dD > 0$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial s} = \int_0^{\frac{Q - \mu^2(r-s)}{2}} f(D) dD > 0$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial g} = \int_Q^\infty f(D) dD > 0$$

$$\text{II) } Q_L^* < \lambda \text{ and } Q_L^* < \frac{\mu^2(r-s)}{2}$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial w} = -1 < 0$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial \mu} = -\frac{4}{\mu^3} \int_0^Q (D-Q) f(D) dD = -\frac{2}{\mu} \int_0^Q \frac{2(D-Q)}{\mu^2} f(D) dD =$$

Substituting from first order condition, we get

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial \mu} = \frac{2}{\mu} \left( \int_0^Q ((r-w)) f(D) dD + (r+g-w) \int_Q^\infty f(D) dD \right) > 0$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial r} = \int_Q^\infty f(D) dD + \int_0^Q f(D) dD = 1 > 0$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial s} = 0$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial g} = \int_Q^\infty f(D) dD > 0$$

III)  $Q_L > \lambda$ , and  $Q_L - \frac{\mu^2(r-s)}{2} < \lambda$ , and  $Q_L \geq \frac{\mu^2(r-s)}{2}$  (only active if  $\lambda$  is finite)

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial w} = -1 < 0$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial \mu} = -\frac{4}{\mu^3} \int_{Q - \frac{\mu^2(r-s)}{2}}^{\lambda} (D-Q)f(D)dD > 0$$

since, in this region,  $D$  is nowhere larger than  $Q$ .

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial r} = \int_{Q - \frac{\mu^2(r-s)}{2}}^{\lambda} f(D)dD > 0$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial s} = \int_0^{Q - \frac{\mu^2(r-s)}{2}} f(D)dD > 0$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial g} = 0$$

IV)  $Q_L > \lambda$ ,  $Q_L - \frac{\mu^2(r-s)}{2} < \lambda$ , and  $Q_L < \frac{\mu^2(r-s)}{2}$  (only active if  $\lambda$  is finite and  $\frac{\mu^2(r-s)}{2} > \lambda$ )

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial w} = -1 < 0$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial \mu} = \frac{4}{\mu^3} (Q - E[D]) > 0$$

since, in this region,  $Q$  is larger than  $E[D]$ .

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial r} = 1 > 0$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial s} = 0$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial g} = 0$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial \sigma} = 0$$

**Proof of Proposition IV**

(a) Note that, under short lead time, optimal effort is  $e_s^* = \frac{\mu(r-w)}{2}$ , a deterministic constant independent of  $D$ , and therefore  $Var(er_s^*) = 0$

On the other hand, long lead time effort at  $t = I$  changes with demand values. Next, we will examine this for the four possible objective functions where the optimum may fall:

I)  $Q_L^* < \lambda$  and  $Q_L^* \geq \frac{\mu^2(r-s)}{2}$

	I	II	III
	$D \leq Q_L^* - \frac{\mu^2(r-s)}{2}$	$Q_L^* > D > Q_L^* - \frac{\mu^2(r-s)}{2}$	$D > Q_L^*$
$e_L^*$	$\frac{\mu(r-s)}{2}$	$\frac{(Q_L^* - D)}{\mu}$	$0$

From the table, it can be seen that effort changes with demand outcomes, and, therefore, in this region,  $Var(er_L^*) > 0$ .

II)  $Q_L^* < \lambda$  and  $Q_L^* < \frac{\mu^2(r-s)}{2}$

	$Q_L^* > D$	$D > Q_L^*$
$e_L^*$	$\frac{(Q_L^* - D)}{\mu}$	$0$

From the table, it can be seen that effort also changes with demand outcomes, and, therefore, in this region,  $Var(e_L^*) > 0$ .

III)  $Q_L > \lambda$ , and  $Q_L - \frac{\mu^2(r-s)}{2} < \lambda$ , and  $Q_L \geq \frac{\mu^2(r-s)}{2}$  (only active if  $\lambda$  is finite)

	I	II
	$D \leq Q_L^* - \frac{\mu^2(r-s)}{2}$	$\lambda > D > Q_L^* - \frac{\mu^2(r-s)}{2}$
$e_L^*$	$\frac{\mu(r-s)}{2}$	$\frac{(Q_L^* - D)}{\mu}$

From the table, it can be seen that effort also changes with demand outcomes, and, therefore, in this region,  $Var(e_L^*) > 0$ .

IV)  $Q_L > \lambda$ ,  $Q_L - \frac{\mu^2(r-s)}{2} < \lambda$ , and  $Q_L < \frac{\mu^2(r-s)}{2}$  (only active if  $\lambda$  is finite and  $\frac{\mu^2(r-s)}{2} > \lambda$ )

	$\lambda > D > 0$
$e_L^*$	$\frac{(Q_L^* - D)}{\mu}$

Here, effort is constant, regardless of demand outcome. Therefore,  $Var(e_L^*) = 0$ .



Therefore, from the analysis for cases 1) to 4),  $Var(e_L^*) \geq 0$ .

(b) Note that, under short lead time, at  $t = 0$ , the manufacturer does not know exactly how much the retailer will order, but, instead, faces an uncertain order quantity whose expected value is:

$$Q_S^* = E[D] + \mu e_s^* = E[D] + \frac{\mu^2(r-w)}{2}$$

Therefore,

$$Var(Q_S^*) = Var(D)$$

On the other hand, under long lead time, at  $t = 0$  the retailer always orders the same constant quantity, and therefore  $Var(Q_L^*) = 0$ .

(c) If we start from the retailer's formulation of long lead time case, essentially the effect of allowing for 0 lead times can be interpreted as:

- (1) moving from a sequential optimization of  $Q$  and  $er$  to a simultaneous one
- (2) removing the constraint  $D + \mu er \leq Q$  present when  $D < Q$

Therefore, the optimal value of retailer profits cannot get worse by allowing zero lead time.

#### **Example of $Q_L^* < E[Q_S^*]$**

By example. Let  $w = r - \varepsilon > 0$ , where  $\varepsilon$  is a very small number,  $s = 0$ , and  $\lambda < \infty$  be the upper limit of the demand distribution.

Then,  $Q_L^* \approx 0$ , in fact, it can be made arbitrarily close to 0 by changing  $\varepsilon$ .

However,

$$E[Q_S^*] \approx E[D]$$

Therefore, for  $\varepsilon$  sufficiently small,

$$Q_L^* \approx 0 < E[Q_S^*] \approx E[D]$$

**Example of  $Q_L^* > E[Q_S^*]$**

By example. Let  $r > 0$ ,  $w = \varepsilon$ , where  $\varepsilon$  is a very small number,  $s = 0$ , and  $\lambda < \infty$  be the upper limit of the demand distribution.

Then, for  $\varepsilon$  sufficiently small,

$$Q_L^* \approx \lambda + \frac{r\mu^2}{2}$$

and

$$E[Q_S^*] \approx E[D] + \frac{r\mu^2}{2}$$

Therefore, for  $\varepsilon$  sufficiently small,

$$Q_L^* \approx \lambda + \frac{r\mu^2}{2} > E[Q_S^*] \approx E[D] + \frac{r\mu^2}{2}$$

**Proof that, for  $\mu = 0$ , the long lead time problem reduces to a newsvendor problem**

Start in the objective function,

$$\begin{aligned}
Z_l^* &= \text{Max}_{Q_l} Z_l = \\
& \int_0^{\frac{\mu^2(r-s)}{2}} \left( (D + \frac{\mu^2(r-s)}{2})(r-w) + (Q_l - (D + \frac{\mu^2(r-s)}{2}))(s-w) - (\frac{\mu^2(r-s)}{2})^2 \right) f(D) dD \\
& + \int_{\frac{\mu^2(r-s)}{2}}^{Q_l} \left( Q_l(r-w) - (\frac{Q_l - D}{\mu})^2 \right) f(D) dD \\
& + \int_{Q_l}^{\lambda} \left( Q_l(r-w) - (D - Q_l)g \right) f(D) dD
\end{aligned}$$

Note that, if  $\mu = 0$ , then the second term disappears, and the objective function becomes

$$\begin{aligned}
Z_l^* &= \text{Max}_{Q_l} Z_l = \\
& \int_0^{Q_l} \left( D(r-w) + (Q_l - D)(s-w) \right) f(D) dD \\
& + \int_{Q_l}^{\lambda} \left( Q_l(r-w) - (D - Q_l)g \right) f(D) dD
\end{aligned}$$

exactly the formulation of a standard newsvendor problem.

### Proof of Proposition V

The proof revolves around observing what happens to  $Q_l^*$  as  $\mu$  grows.

#### a) Distribution with finite upper bound

$$I) Q_l \leq \lambda \text{ and } Q_l \geq \frac{\mu^2(r-s)}{2}$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_l^* \partial \mu} = - \frac{4}{\mu^3} \int_{\frac{\mu^2(r-s)}{2}}^{Q_l} (D - Q_l) f(D) dD = \frac{2}{\mu} \int_{\frac{\mu^2(r-s)}{2}}^{Q_l} -\frac{2}{\mu^2} (D - Q_l) f(D) dD$$

We are unable to sing this crosspartial derivative. However, this is no obstacle.

a) If  $\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial \mu} < 0$ , then, as  $\mu$  grows,  $Q_L^*$  becomes smaller, and  $\frac{\mu^2(r-s)}{2}$  larger. Eventually, then,  $Q_L < \frac{\mu^2(r-s)}{2}$ , and the objective function in region II applies.

b) If  $\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial \mu} > 0$  is positive, then as  $\mu$  grows, so does  $Q_L^*$ .

b1) If  $Q_L^*$  grows with  $\mu$  at a slower rate than  $\frac{\mu^2(r-s)}{2}$ , then eventually  $Q_L < \frac{\mu^2(r-s)}{2}$ , as, again, the objective function from region II applies.

b2) If, on the other hand,  $Q_L^*$  grows faster than  $\frac{\mu_A^2(r_A-s_A)}{2}$ , then eventually we'll have  $Q_L > \lambda$ , and the objective function in region III applies.

Therefore, no matter what the sign of  $\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial \mu}$  is in this region, as  $\mu$  grows we would end in either regions II or III.

II)  $Q_L \leq \lambda$  and  $Q_L < \frac{\mu^2(r-s)}{2}$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial \mu} = \frac{2}{\mu} \left( \int_0^Q ((r-w)) f(D) dD + (r+g-w) \int_Q^\infty f(D) dD \right) > 0$$

Therefore,  $Q_L^*$  will grow with  $\mu$  until  $Q_L > \lambda$ , moving us into region III..

III)  $Q_L > \lambda$ , and  $Q_L - \frac{\mu^2(r-s)}{2} < \lambda$ , and  $Q_L \geq \frac{\mu^2(r-s)}{2}$  (only active if  $\lambda$  is finite)

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial \mu} = - \frac{4}{\mu^3} \int_{Q - \frac{\mu^2(r-s)}{2}}^{\lambda} (D-Q) f(D) dD > 0$$

Therefore,  $Q_L^*$  will grow with  $\mu$ . But will  $\frac{\mu^2(r-s)}{2}$  grow faster than  $Q_L^*$ ?

$$\frac{\partial E[\frac{\mu^2(r-s)}{2}]}{\partial Q} = \mu(r-s)$$

$$\text{We know that } \frac{\partial^2 E[\pi_L^{ret}]}{\partial^2 Q} = \int_{Q-\frac{\mu^2(r-s)}{2}}^{\lambda} -(\frac{2}{\mu^2})f(D)dD$$

Therefore ,

$$\frac{dQ_L^*}{d\mu} = -\frac{\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial \mu}}{\frac{\partial^2 E[\pi_L^{ret}]}{\partial^2 Q_L^*}} = -\frac{2}{\mu} \int_{Q-\frac{\mu^2(r-s)}{2}}^{\lambda} (D-Q)f(D)dD = \frac{2}{\mu} [Q - \int_{Q-\frac{\mu^2(r-s)}{2}}^{\lambda} (D)f(D)dD]$$

so, as  $\mu$  grows,  $Q_L^*$  grows slower and slower, but  $\frac{\mu^2(r-s)}{2}$  grows faster and faster. Eventually,

$Q_L^* < \frac{\mu^2(r-s)}{2}$ , thus making region IV the relevant formulation.

IV)  $Q_L > \lambda$ ,  $Q_L - \frac{\mu^2(r-s)}{2} < \lambda$ , and  $Q_L < \frac{\mu^2(r-s)}{2}$  (only active if  $\lambda$  is finite and  $\frac{\mu^2(r-s)}{2} > \lambda$ )

$$\begin{aligned} \frac{\partial E[\pi_L^{ret}]}{\partial Q} &= \int_0^{\lambda} ((r-w) + 2\frac{(D-Q)}{\mu^2})f(D)dD \\ &= (r-w) + \frac{2}{\mu^2} \int_0^{\lambda} (D-Q)f(D)dD \\ &= (r-w) + \frac{2}{\mu^2} [\int_0^{\lambda} (D)f(D)dD - Q] \\ &= (r-w) + \frac{2}{\mu^2} [E[D] - Q] \end{aligned}$$

But, setting this derivative equal to zero and solving yields

$$Q_S^* = E[Q_L^*] = E[D] + \frac{(r-c)\mu^2}{2}.$$

Therefore, if the distribution has a finite upper limit,  $\exists$  some  $\mu'$  such that if  $\mu > \mu'$ , then

$$Q_L = E[Q_S].$$

This completes the proof.

### b) Distribution with infinite upper bound

$$1) Q_L \geq \frac{\mu^2(r-s)}{2}$$

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial \mu} = -\frac{4}{\mu^3} \int_{Q - \frac{\mu^2(r-s)}{2}}^Q (D-Q)f(D)dD = \frac{2}{\mu} \int_{Q - \frac{\mu^2(r-s)}{2}}^Q -\frac{2}{\mu^2}(D-Q)f(D)dD$$

As mentioned, we are unable to sing this crosspartial derivative. Again, this is no obstacle.

a) If  $\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial \mu} < 0$ , then, as  $\mu$  grows,  $Q_L^*$  becomes smaller, and  $\frac{\mu^2(r-s)}{2}$  larger. Eventually, then,  $Q_L < \frac{\mu^2(r-s)}{2}$ , and the objective function in region II applies.

b) If  $\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial \mu} > 0$  is positive, then as  $\mu$  grows, so does  $Q_L^*$ .

b1) If  $Q_L^*$  grows with  $\mu$  at a slower rate than  $\frac{\mu^2(r-s)}{2}$ , then eventually  $Q_L < \frac{\mu^2(r-s)}{2}$ , as, again, the objective function from region II applies.

b2) If, on the other hand,  $Q_L^*$  grows faster than  $\frac{\mu^2(r-s)}{2}$ , then in the objective function

$$\int_0^{Q_L - \frac{\mu^2(r-s)}{2}} \left( (D + \frac{\mu^2(r-s)}{2})(r-w) + (Q_L - (D + \frac{\mu^2(r-s)}{2}))(s-w) - (\frac{\mu^2(r-s)}{2})^2 \right) f(D) dD \rightarrow$$

$$\int_0^{\infty} \left( (D + \frac{\mu^2(r-s)}{2})(r-w) + (Q_L - (D + \frac{\mu^2(r-s)}{2}))(s-w) - (\frac{\mu^2(r-s)}{2})^2 \right) f(D) dD$$

which means that

$$\int_{Q_L - \frac{\mu^2(r-s)}{2}}^{Q_L} \left( Q_L(r-w) - (\frac{Q_L - D}{\mu})^2 \right) f(D) dD \rightarrow 0$$

and

$$\int_{Q_L}^{\infty} \left( Q_L(r-w) - (D - Q_L)g \right) f(D) dD \rightarrow 0$$

Now, notice that in

$$\int_0^{\infty} \left( (D + \frac{\mu^2(r-s)}{2})(r-w) + (Q_L - (D + \frac{\mu^2(r-s)}{2}))(s-w) - (\frac{\mu^2(r-s)}{2})^2 \right) f(D) dD =$$

the constraint on  $Q_L$  will bind (since  $s < w$ ), thus the objective function will be

$$\int_0^{\infty} \left( (D + \mu \frac{Q_L - D}{\mu})(r-w) + (Q_L - (D + \mu \frac{Q_L - D}{\mu}))(s-w) - (\frac{Q_L - D}{\mu})^2 \right) f(D) dD$$

This is exactly the same objective function as region IV in our bounded distribution example, and we proved above that in this region  $Q_L = E[Q_S]$ .

Therefore, in this case the solution to the max problem approaches  $Q_L = E[Q_S]$ , or, in other words,  $\exists$  some  $\mu'$  such that if  $\mu > \mu'$ , then  $Q_L = E[Q_S]$ .

$$\text{II) } Q_L < \frac{\mu^2(r-s)}{2}$$

we know that

$$\frac{\partial^2 E[\pi_L^{ret}]}{\partial Q_L^* \partial \mu} = \frac{2}{\mu} \left( \int_0^Q ((r-w)) f(D) dD + (r+g-w) \int_Q^\infty f(D) dD \right) > 0$$

Therefore, as  $\mu$  grows,  $Q_L^*$  grows, and in the objective function

$$\begin{aligned} & \int_0^{Q_{II}} \left( Q_{II}(r-w) - \left( \frac{Q_{II}-D}{\mu} \right)^2 \right) f(D) dD \\ & \rightarrow \int_0^\infty \left( Q_L(r-w) - \left( \frac{Q_L-D}{\mu} \right)^2 \right) f(D) dD \end{aligned} \quad (ZZ)$$

$$\int_{Q_{II}}^\infty (Q_{II}(r-w) - (D-Q_{II})g) f(D) dD \rightarrow 0$$

Again, (ZZ) is exactly as the objective function in region IV, and, therefore, its solution is the same as in the short lead time case. This means that, in this case, the solution to the max problem also approaches  $Q_L = E[Q_S]$ , or, in other words,  $\exists$  some  $\mu'$  such that if  $\mu > \mu'$ , then  $Q_L = E[Q_S]$ .

This completes the proof.

### Proof of Proposition VI

$$\frac{\partial \pi_s^{man}}{\partial w} = E[D] + (c+r-2w) \frac{\mu^2}{2}$$

and

$$\frac{\partial^2 \pi_s^{man}}{\partial^2 w} = -\mu^2$$



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## Notes

<sup>i</sup> Throughout this paper, we will adopt the common convention of making the upstream party (i.e. the manufacturer) female, and the downstream party (i.e. the retailer) male.

<sup>ii</sup> A minor difference is that while Iyer and Bergen model a Normal distribution, we generalize part of our insights to a generic distribution.

<sup>iii</sup> If effort was exerted before demand is realized, then because demand generated by effort is deterministic (as it will become clear later), the part of the quantity bought by the retailer that is a response to effort induced demand would be the same under long and short lead times.

<sup>iv</sup> Insights could be generalized to any generic effort cost function  $V(e)$ , with  $V(0) = 0$ ,  $V'(\cdot) \geq 0$  and  $V''(\cdot) > 0$ .

<sup>v</sup> This assumption is made purely for to simplify calculations. As we mentioned earlier, the essence of the paper is the comparison of manufacturers profits under short and long lead times. A demand dependent cost of effort function would not change the papers' insights, since demand itself is exogenous and the same for short and long lead times. On the other hand, the cost of effort should not change with lead time, for the same reasons that made us assume that the manufacturer's lead time reduction is free.

<sup>vi</sup> This could be interpreted as if the retailer observed some perfectly informative signal of demand at  $t = I$ . This, however, need not be the case. As it will become clearer later in this paragraph, for short lead times we could assume that delivery happens instantly as customers materialize. The reason for this assumption is explained later in this paper.

<sup>vii</sup> At this stage,  $Q$  is sunk, and therefore it is not necessary to include it in the retailer's profit function. We chose to do so in all cases for clarity of the exposition.

<sup>viii</sup> It can be shown that, even under short lead times, channel profits under a price only contract are less than first best –i.e. an integrated decision maker would achieve higher profits–, due to double marginalization, as in Spengler (1950), Pasternack (1985).

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