WHEN IS VENDOR MANAGED INVENTORY GOOD FOR THE RETAILER? IMPACT OF RELATIVE MARGINS AND SUBSTITUTION RATES

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Abstract

When customers cannot find a particular item at a retailer because it is out of stock, they are likely, with some probability, to switch to a substitute product from another manufacturer at the same store. Analyzing a two-product full substitution case, this paper examines two beliefs argued in the literature: (1) that, under Vendor Managed Inventory (VMI), the retailers would benefit because manufacturers would increase stocking quantities to avoid losing sales to a competitor and (2) that substitution benefits retailers who make a sale regardless. We find that the first proposition, while appealing, is simply not true for many cases. We also find that the second proposition does not hold for a wide number of cases. We contribute to the understanding of the inherent tradeoffs involved in deciding to use RMI or VMI in the presence of competing, substitute products.

Keywords

Newsvendor, substitution, Vendor Managed Inventory, retailer margin, manufacturer margin

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1. Introduction

There has been a lot of recent work on Vendor Managed Inventory (VMI). Some authors have argued that VMI can allow manufacturers to obtain better information, -*e.g.* Clark and Hammond (1997), Cachon and Fisher (1997)-, others that VMI can allow manufacturers to "pool" inventory or consolidate shipping, -*e.g.*, Cheung and Lee (2002). Others have focused on the relationship between a single manufacturer and a retailer, and identified conditions under which VMI would achieve better total profits for the supply chains, regardless of how parties allocate these –*e.g.* Kraiselburd, Narayanan and Raman (2004). In this paper, instead, we take the retailer's perspective, considering both cases when products have no competitors and when substitution is possible, and pose the question: when is VMI good for the retailer?

When two or more products are possible substitutes, customers facing an out-of-stock of their most preferred item may substitute for an alternative product, buy later, or walk out the store and either buy from a competitor or simply not purchase the item. Corsten, Bharadwaj and Gruen (2002), a report that consolidates results from 52 studies across different retailers in different parts of the world surveying a cumulative total of 71,000 consumers in 32 different consumer goods categories, state than, when facing an out of stock, an average of 45% of customers substitute to a different product, 40% of customers either buy at a different store or do not buy the product at all, and 15% of customers delay the purchase. In this scenario, it has been argued that under VMI the retailers would benefit from manufacturers increasing stocking quantities as a result of competition because a substitute sale may still be a lost sale for the manufacturer–*e.g.* Mishra and Raghunathan (2004)-. In this paper we qualify of this statement: whether manufacturers will stock more under VMI than under Retailer Managed Inventory (RMI) mostly depends on three factors (a) each manufacturer's margins, (b) the retailer's margins on each product, and (c) the degree of substitution to and from each product. Therefore, a retailer considering whether to ask for VMI from its manufacturers must be fully aware of the intricacies of his[i] situation. Moreover, depending on the relative margins of each product, even if a manufacturer does stock more, we show that this could potentially *reduce* the retailer's profits. As a consequence of this, we find that, depending on the situation, retailers may prefer doing RMI for both products, RMI for one and VMI for another, and VMI for both products.

If the retailer's margins are the same as the manufacturers', and both products are symmetric, then it will be to the retailer's advantage to switch to VMI. If the retailer makes high margins on both products, but the manufacturers do not (something to be expected, for example, the products are the retailer's own brand), then it is likely that switching to VMI will make the quantities stocked for both products drop, thus the manufacturer is likely to benefit from managing the inventory of both products –*i.e.*, doing RMI for both products-. This intuition applies to both exclusive products and products that are subject to substitution.

If, on the other hand, both of the manufacturers' margins are high but the retailer's are low, then it may be appropriate for the retailer to give stocking decisions to the manufacturers of both products –*i.e.*, doing VMI for both products. Again, the logic here would apply both to exclusive products and products facing substitution.

If some customers do switch to a substitute product when facing an out-of-stock, the retailer makes more margin on one product than on another, (which is again possible, for example, if one of the products is the retailer's own brand), and the manufacturers' margins are larger than the retailer's for both products, then doing VMI for both products may be less desirable for the retailer than giving decision rights to the manufacturer of the more lucrative product, but keeping stocking decisions to the less lucrative product –*i.e.*, doing VMI for one product, and RMI for the other. The reason for this is that it may be beneficial for the retailer if *less* is stocked of the less lucrative product, thus indirectly transferring some demand to the more lucrative product. However, if the manufacturer of the less lucrative product makes the stocking decisions, she will maximize her profits by stocking *more* because she does not care for transfer sales –that go to her competitor-.

Finally, if substitution is possible, the retailer makes similar margins on both products, but the manufacturer of one product makes larger margins than the retailer, and the manufacturer of the other product makes less margins than the retailer, it may be optimal to transfer decision rights to the manufacturer with higher margins (than the retailer's), but keeping decision rights of the product where the retailer makes higher margins –*i.e.*, letting the party making the higher margin make the decisions-. Note how, in the last two examples, the rationale for giving stocking decisions to one of the manufacturers is the opposite: while in the former case, doing VMI for the product with low retailer margins could damage the retailer because the manufacturer would stock *more* than the retailer when *less* was desirable, in the latter case stocking decisions are allocated to the party that would stock the *most*.

Throughout the paper, we have assumed that retailer and manufacturer margins are exogenous, that is, that they do not change as a result of either VMI or RMI being implemented. This is consistent with at least some cases of VMI implementations. For example, both in Barilla -Hammond, (1994)- and Campbell Soup - Clark and McKenney (1994), Clark and Hammond (1997), Cachon Fisher (1997)-- VMI was combined with reducing the number of promotions, but the average price offered to retailers remained the same). Another possible application is the newspaper industry: most newspapers face few substitutes on each city: in fact, in many big metropolitan areas there are mostly two competitors. Both retail and wholesale prices change very rarely. However, when a change does happen, arguments about stocking quantities are common. For example, when, around 1909 in Chicago, Hearst decided to lower the newsvendors margins from 60% to 40%, the newsvendors either reduced their stocking quantities dramatically (attempting to "switch" customers to the newspapers that were more lucrative for them), or refused to carry the *American* –Hearst' newspaper- altogether, Bekken (2000). At that time, the newspaper reacted by attempting to force the newsvendors into VMI, in some cases actually resorting to coercion and violence. In our recent work with a prominent newspaper in one of the largest US metropolitan areas which is much cheaper than the competition, when the newsvendors make stocking decisions, they carry much less than when the newspapers make them. In fact, the newspaper is in a constant argument over who decides the stocking quantities. This resulted in switching back and forth from VMI to RMI numerous times at its more than 6000 newsvendors, while both retail and wholesale prices have remained constant for years. In this industry, the newspaper's margins are given by advertising (which is proportional to circulation), as well as the actual sale price. Thus, although a newspaper may choose to sell for a low price –leaving small margins for the newsvendors-, its margins may be large because of the impact of circulation in advertising.

In addition, it has long been thought that substitution benefits retailers: indeed, we have seen in previous papers numerous cases when the retailer makes more profits as more customers are willing to substitute for another product when their favorite is out of stock. In this paper, however, we find that this is not necessarily true: if more customers are willing to substitute a product that leaves high margins to the retailer for a product that leave low margins to the retailer, the retailer may very well be worse off. We find that this result to be more likely under VMI than under RMI.

In our analysis we consider the following possibilities: (a) either substitution is possible or it is not, (b) either customers who face an out of stock do not purchase the product at the retailer (lost sales case), or simply delay the purchase (backorders case). We analyze the pure backorders case and the pure lost sales case to provide insights about what conclusions can be generalized to both scenarios and what cannot (the most general case that assumes both lost sales and backorders are possible at the same time has not been studied because of tractability limitations).

The rest of the paper is organized as follows: Section 2 provides a literature review. Section 3 provides the main model and demand structure: 3.1 develops the expressions for sales, inventory, shortage and transfer sales for both the backorders and lost sales cases, 3.2 formulates the Retailer Managed Inventory case, 3.3 the Vendor Managed Inventory case, 3.4 the Mixed cases of Vendor Managed and Retailer Managed Inventory . Section 4 analyzes the full backorders case: 4.1 looks at RMI for both products, 4.2, VMI for both products, 4.3., RMI for one but VMI for the other, and 4.4 compares the three cases. Section 5 extends the analysis to the lost sales case. Again, 5.1 studies RMI for both products, 5.2, VMI for both products, 5.3. RMI for one product and VMI for another, and 5.4. compares the three cases. Section 6 concludes the paper. Appendix 1 includes figures, and Appendix 2, proofs.

2. Literature Review

A number of papers model situations where consumers would consider a substitute product when their preferred product is out of stock. Some models assume one-way substitutability, as in Bassok, Anupindi and Akella (1999). Others consider two-way substitutability: among them, Parlar and Goyal (1984) and Parlar (1988) are two early examples of two product cases. Noonan (1995) formulates the general substitution based newsvendor problem with lost sales for n-substitutability, and Netessine and Rudi (2003) re-formulate the problem using a simpler notation and achieve further results. All of the papers in this stream of literature assume lost sales (*i.e.*, when faced with a stockout, customers either buy a substitute product or simply leave). A general closed form solution for this problem has not been found, and many papers resort to approximations, heuristics, or numerical solutions -for example Rajaram and Tang (2000)-. Typically, these papers consider a single stage supply chain.

Some authors have compared VMI and RMI to a first best solution, identifying conditions under which VMI perform better than RMI for the whole supply chain. Cohen Kulp (2002) argues that VMI would be better if enough information can be revealed to the manufacturer by the retailer. Other papers argue that VMI could mitigate competition among retailers by centralizing stocking decisions with the manufacturer, for example, Cachon (2001). Çetinkaya and Lee (2000) argue that VMI allows shipment consolidation. Fry, Kapuscinski and Lennon Olsen (2001) and Bernstein, Chen and Federgruen (2002) are other examples that analyze VMI.

However, the only two other papers that we are aware of that compare VMI to RMI in the presence of substitute products are Kraiselburd, Narayanan and Raman (2004), and Mishra and Raghunathan (2004). Our paper differs from Kraiselburd, Narayanan and Raman (2004) in a number of ways: while in their model the presence of uncontractible manufacturer’s efforts is key to the difference between RMI and VMI, our model does not consider any effort; these three authors compare a branded product to a generic, retail brand, we model a more general situation that includes two different manufacturers; these authors consider the lost sales case, we model both lost sales and backorders. In addition, their very definition of VMI differs from ours: in Kraiselburd, Narayanan and Raman (2004), VMI consists of a contract where the retailer gives the manufacturer decision rights over quantities, and makes her responsible for holding costs, but the manufacturer does not charge the retailer any wholesale price; instead, the manufacturer pays the retailer a flat fee, or “slotting allowance”. In this paper, VMI also consists of a contract where the retailer gives the manufacturer decision rights over quantities, and makes her responsible for holding costs, however, the manufacturer keeps on charging the retailer a wholesale price, and we do not consider lump-sum money transfers. This is precisely the definition of VMI used in Mishra and Raghunathan (2004). Although it can be considered an extension of their work, our paper also differs significantly from Mishra and Raghunathan (2004): while their paper focuses on two symmetric products under full backorders, we extend the analysis to asymmetric products and the lost sales case. These two extensions are not trivial: we show that most of the results of the symmetric products case with full backorders do not generalize to asymmetric products or to lost sales models: (a) in Mishra and Raghunathan (2004), an increase in the proportion of customers willing to substitute one product for the other always leads to higher profits for the retailer, while we show that this is no longer necessarily the case with lost sales or asymmetric products, and (b) in Mishra and Raghunathan (2004), the retailers profits under VMI are at least as large as under RMI, while we show that this is not necessarily true for asymmetric cases, or assuming lost sales.

While both Kraiselburd, Narayanan and Raman (2004), and Mishra and Raghunathan (2004) assume endogenous wholesale prices, in this paper we do not. In the former, the authors are able to study the endogenous wholesale price case because one of the products in their model is a retailer brand, and, thus, does not have a transfer price; in the latter, both products are assumed to be symmetric, which significantly simplifies the analysis: either the retailer does VMI for both products, or none, either substitution for both products increases or none, etc. In our paper, the retailer may choose to do VMI either with one of both products, and asymmetry plays a central role: for example, when substitution from one product to another changes but not vice versa, the retailer may be worse off. Thus, by restricting our analysis to the endogenous margins case, we are able to relax other assumptions.

3. Model Development and Demand Structure

This section describes assumptions that relate to product demand, revenues, inventory replenishment, the cost of production, and inventory holding costs. We model the interaction between two manufacturers and a retailer. Manufacturer 1 produces product 1, Manufacturer 2, product 2, but the retailer sells both products. We assume the retail price for product i ($i \varepsilon \{1,2\}$) during the period of interest to be p_i. It costs each manufacturer c_i to make each unit of her product, and the manufacturers sell to the retailer at a cost of w_i per unit. p_i and w_i are exogenously determined. Stocking level decisions are made each period before demands are realized. Finally, we will assume that the problem is an infinite-horizon average profit maximization problem, where, at the end of each period, the products can be held until the next period if a holding cost is paid –*i.e.* we are assuming a variation of an infinitely repeated newsvendor problem-. Let h_{si} be holding cost per unit of product i per unit of time associated with whoever physically holds the stock (associated with space, etc.),

h_{ci} be the holding cost per unit of product i per unit of time associated with whoever owns the stock (opportunity cost of capital, obsolescence, etc.), and $h_i = h_{ci} + h_{si}$. Also, let $p_i > w_i > h_i$.

We model demand as consisting of two streams: "first choice demand" for a product consists of demand from those consumers that preferred this product, and "second choice demand" consists of demand from consumers who want to purchase this product because their preferred product was stocked out. First choice demand for product i, is denoted by d_i, a random variable with a strictly increasing and continuous cumulative distribution. The retailer carries a second product, product j ($j \in \{1,2\}, i \neq j$,), which could substitute for product i. Some fraction α_{ij}, of customers who experience a stockout of product i will switch to this substitute product at the same retail store. We will assume that j's initial demand was satisfied first, just as, among others, Parlar and Goyal (1984), Noonan (1995), Netessine and Rudi (2003), and Kraiselburd, Narayanan and Raman (2004).Let q_i represent the stocking quantity for product i.

There are two possible assumptions about what happens to (a) the $(1-\alpha_{ij})$ consumers who, when experiencing a stockout of their preferred product, i, will not switch to product j, and

(b) those consumers that, when facing a stockout of their favorite product, *i*, did intend to switch to product *j* but did not find any stock of product *j* either (because product *j* also stocked out): either customers in cases (a) and (b) *backorder* product *i,* and the retailer incurs a goodwill loss because of this, or, those customers simply walk out, and the retailer incurs a lost sale. In this paper, we will consider both scenarios, and investigate the consequences of assuming either way.

Note that, either in the backorder or the lost sale case, a lost sale for the manufacturer is not necessarily a lost sale for the retailer, because the demand that transfers from product *i* to product *j* is lost for the manufacturer of product *i* but not to the retailer. However, there may be a goodwill loss to the retailer for customers that have to settle for their second best. Let parameter t_{ij}, be defined as the dollar per unit goodwill cost of a sale of product *j* that originated from customers who initially preferred product *i*.

In addition, the manufacturer's per unit goodwill loss of being short needs not be the same as the retailer's. Let s_{ri} be the retailer's goodwill loss on a lost sale of good *i,* and s_{mi} be the manufacturer's goodwill loss on a lost sale of good *i*. Later in the paper, we will be making both parameters equal to achieve a *ceteris paribus* comparison between VMI with RMI. However, at this stage, we would like to formulate the problem in the most general way.

Finally, given the comments above, a general problem formulation would require separate expressions for the retailer's and the manufacturer's shortage. This will be illustrated in detail when explaining the expressions for sales, inventory, and shortage.

Fortunately, as it will become clear later, some assumptions made in the spirit of a *ceteris paribus* comparison between VMI with RMI will eliminate the need to keep separate expressions, thus simplifying the problem somewhat.

3.1 Expressions for Sales, Inventory, Shortage, and Transfer sales.

The easiest way to formulate all the possible outcomes is to divide the problem in six regions. (see Appendix 1, Figure 1 for a graph that illustrates these regions).

The six possible regions are given by:

(*i*) $d_1 \leq q_1$ and $d_2 \leq q_2$, *i.e.*, neither product stock out.

	Lost sales Scenario	Backorders Scenario
$Sales_i$	d_i	d_i

Inventory$_i$	$q_i - d_i$	$q_i - d_i$
Shortage$_i$	0	0

(ii) $d_1 > q_1$ and $d_2 > q_2$, *i.e.*, both products stock out on originating demand.

	Lost sales Scenario	Backorders Scenario
Sales$_i$	q_i	d_i
Inventory$_i$	0	0
Shortage$_i$	$d_i - q_i$	$d_i - q_i$

(iii) $d_1 > q_1$, $d_2 \le q_2$ and $\alpha_{12}(d_1 - q_1) \le q_2 - d_2$, *i.e.*, product 1 stocks out on originating demand, product 2 does not, and indirect substitution demand is not enough to stock out product 2.

	Lost sales Scenario	Backorders Scenario
Sales$_1$	q_1	$d_1 - (d_1 - q_1)(\alpha_{12})$
Sales$_2$	$d_2 + \alpha_{12}(d_1 - q_1)$	$d_2 + \alpha_{12}(d_1 - q_1)$
Inventory$_1$	0	0
Inventory$_2$	$q_2 - [d_2 + \alpha_{12}(d_1 - q_1)]$	$q_2 - [d_2 + \alpha_{12}(d_1 - q_1)]$
Shortage$_1$ for retailer	$(d_1 - q_1)(1 - \alpha_{12})$	$(d_1 - q_1)(1 - \alpha_{12})$
Shortage$_1$ for manufacturer	$(d_1 - q_1)$	$(d_1 - q_1)$

Transferred$_1$ and sold, for retailer	$(d_1 - q_1)(\alpha_{12})$	$(d_1 - q_1)(\alpha_{12})$
Shortage$_2$	0	0

(iv) $d_1 > q_1$, $d_2 \le q_2$ and $\alpha_{12}(d_1 - q_1) > q_2 - d_2$, *i.e.*, product 1 stocks out on originating demand, product 2 does not, but indirect substitution demand is enough to stock out product 2.

	Lost sales Scenario	Backorders Scenario
Sales$_1$	q_1	$d_1 - (q_2 - d_2)$
Sales$_2$	q_2	$d_2 + (q_2 - d_2) = q_2$
Inventory$_1$	0	0
Inventory$_2$	0	0
Shortage$_1$ for retailer	$(d_1 - q_1) - (q_2 - d_2)$	$(d_1 - q_1) - (q_2 - d_2)$
Shortage$_1$ for manufacturer	$(d_1 - q_1)$	$(d_1 - q_1)$
Transferred$_1$ and sold, for retailer	$(q_2 - d_2)$	$(q_2 - d_2)$
Shortage$_2$	0	0

Note: for the retailer,

$$Shortage_1 = (d_1 - q_1)(1 - \alpha_{12}) + Transferred_1 \text{ but not sold}$$

$$Transferred_1 \text{ and sold} = (q_2 - d_2)$$

$$Transferred_1 \text{ but not sold} = [\alpha_{12}(d_1 - q_1) - (q_2 - d_2)]$$

This is true assuming that 2's initial demand was satisfied first, just as, among others, Parlar and Goyal (1984), Noonan (1995), Netessine and Rudi (2003), and Kraiselburd, Narayanan and Raman (2004) assumed.

$Shortage_2 = 0$

Again, assuming that 2's initial demand was satisfied.

(v) $d_1 \le q_1$, $d_2 > q_2$ and $\alpha_{21}(d_2 - q_2) \le q_1 - d_1$, *i.e.*, product 2 stocks out on originating demand, product 1 does not, and indirect substitution demand is not enough to stock out product 1.

Analogous to (iii)

(vi) $d_1 \le q_1$, $d_2 > q_2$ and $\alpha_{21}(d_2 - q_2) > q_1 - d_1$, *i.e.*, product 2 stocks out on originating demand, product 1 does not, but indirect substitution demand is enough to stock out product 1.

Analogous to (iv).

Using the tables constructed for cases (*i*) to (vi), expressions for sales, inventory, shortage, and transfer costs, can be formulated, but will be omitted in the interest of brevity.

At this point, we will assume that:

a) for the full backorder case, $s_{ri} = s_{mi} = t_{ji} = s$, in which case, the expressions of shortage cost and quantities for the retailer are equal to those of the manufacturer. As mentioned earlier, we do this for two reasons: (1) it makes the problem more tractable, and (2) is appropriate for a *ceteris paribus* comparison of VMI and RMI: it would be relatively simple to state that VMI and RMI are different based on the difference in stockout cost that a retailer and a manufacturer experience. We intend to show that there are differences between these two *even* if goodwill losses where equal. This is what Mishra and Raghunathan (2004) implicitly assume for the backordered case.

b) for the lost sales case, $s_{ri} = s_{mi} = t_{ij} = 0 \;\; i \in \{1,2\}, i \ne j$, which represents a special case of Noonan's (1994) formulation, and is mathematically equivalent to Netessine and Rudi's (2004) model . This later assumption makes the more complicated lost sales case more tractable, although results can be generalized to the case where goodwill loss is non zero. Note that assuming this in the full backorder case would lead to extreme results because not meeting demand would be "free". However, in the lost sales case, the retailer/manufacturer loses sales when demand is not met.

Next, we will define three possible scenarios: either the retailer chooses to use Retailer Managed Inventory for both products (RMI), or Vendor Managed Inventory for both products (VMI), or RMI for product *i* product and VMI for product *j* (RMI_iVMI_j).

3.2 Retailer Managed Inventory for both products

Under retailer managed inventory (RMI), the retailer decides stocking quantities and pays the holding cost on inventory left at the end of each period (see Appendix 1, Figure 2 for a graphic depiction of this case).

The retailer's problem is:

$$\underset{q_i,q_j}{Max} \sum_{i\in\{1,2\},i\neq j} (p_i - w_i)S_i - h_i I_i - s_i Sh_i$$

where:

S_i, I_i, and Sh_i represent the integral over all possible cases of the sales, inventory, and shortage expressions formulated in section 3.1. Note that $s_i = 0$ for the lost sales case.

Each manufacturer's profits are given by:

$$(w_i - c_i)S_i - s_i Sh_i,\ i \in \{1,2\}.$$

i.e., the manufacturer pays the same goodwill cost of a shortage (as discussed earlier).

3.3 Vendor Managed Inventory for both products

Under vendor managed inventory (VMI), each manufacturer decides stocking quantities and pays the holding cost on inventory left at the end of each period (see Appendix 1, Figure 3 for a graphic depiction of this case).

If retailers pay for the physical cost of holding inventory (opportunity cost of space, etc.), then the retailer's profit function is:

$$\sum_{i\in\{1,2\},i\neq j} (p_i - w_i)S_i - h_{si} I_i - s_i Sh_i$$

In this paper, however, we will assume here that the retailer can charge the manufacturers for this cost, thus transferring the full holding cost to the manufacturers. If this was not the case, then the transfer of decision rights to the manufactures because of a "subsidized holding cost" would have the effect of making the manufacturer want to stock more under VMI. In this paper, as in others, we intend to see whether VMI and RMI differ even if holding costs where equal for manufacturers and retailers. This is exactly the (implicit) assumption made by Kraiselburd, Narayanan, and Raman (2004) for a lost sales case, and Misrha and Raghunathan (2004) for a full backorder case. In any case, as it will become clear later, none of the paper's insights change with the allocation of holding costs[ii].

Thus, the retailer's profits will be given by:

$$\sum_{i\in\{1,2\}, i\neq j} (p_i - w_i)S_i - s_i Sh_i$$

And manufacturer i 's $\in \{1,2\}$ problem is:

$$\underset{q_i}{Max}\ (w_i - c_i)S_i - h_i I_i - s_i Sh_i$$

3.4 Mixed cases of Vendor Managed and Retailer Managed Inventory

The retailer may choose to use vendor managed inventory (VMI) with one product, but keep using retailer managed inventory for the other. In this scenario, as expected, one manufacturer decides stocking quantities and pays the holding cost on inventory left at the end of each period for one product, and the retailer decides stocking quantities and pays the holding cost on inventory left at the end of each period for the other product (see Appendix 1, Figure 4 for a graphic depiction of this case).

Let the retailer use RMI for product i, and VMI for product j. In this scenario, the retailer solves:

$$\underset{q_i}{Max}\ -h_i I_i + \sum_{i\in\{1,2\}, i\neq j} (p_i - w_i)S_i - s_i Sh_i$$

while the manufacturer under VMI solves

$$\underset{q_j}{Max}\ (w_j - c_j)S_j - h_j I_j - s_j Sh_j$$

4. Analysis of the Full Backorder Case

Next, we will study the full backorder case, comparing retailer profits and quantities stocked under RMI ,VMI, and RMI_iVMI_j, as well as some key comparative statics.

4.1 RMI for both products

Proposition *1*

For a given w_i, w_j, p_i, p_j, and full backorders, under RMI, $i \in \{1,2\}, i \neq j$,

(*i*) if $(p_i - w_i) \leq (p_j - w_j) + h_j$, the retailer's profits are (weakly) increasing in α_{ij}, *i.e.,* $\frac{\partial \pi_R^{*RMI}}{\partial \alpha_{ij}} \geq 0$

(ii)) if $(p_i - w_i) \geq (p_j - w_j) + h_j$, the retailer's profits are (weakly) decreasing in α_{ij}, *i.e.,* $\frac{\partial \pi_R^{*RMI}}{\partial \alpha_{ij}} \leq 0$.

Note: all proofs are in the Appendix.

One can build intuition about Proposition 1's results by noting that the marginal contribution of an increase of α_{ij}, yields two separate components:

(1) more sales and less holding of product *j*: $(p_j - w_j) + h_j$

(2) less sales of product *i*, (because α_{ij} customers will not backorder product *i* if they can transfer to product *j*): $-(p_i - w_i)$.

Therefore, as long as (2) is less than (1), an increase in α_{ij} increases profits for the retailer. Proposition 1 is compatible with Mishra and Raghunathan (2004) who, for symmetric cases, find that $\frac{\partial \pi_R^{*RMI}}{\partial \alpha} \geq 0$.

4.2 VMI for both products

Proposition *2*

For a given w_i, w_j, p_i, p_j, and full backorders, under VMI, $i \in \{1,2\}, i \neq j$, then:

(a) an increase in α_{ji}, will make q_i^{*VMI} (weakly) increase but q_j^{*VMI} decrease (weakly),

(b) if $(p_i - w_i) \geq (p_j - w_j)$, and $-[(p_j - w_j) - (p_i - w_i)] \frac{\partial S_j}{\partial q_j} \geq s_j \frac{\partial Sh_j}{\partial q_j}$, then the retailer's profits are (weakly) increasing in α_{ji}, *i.e.,* $\frac{\partial \pi_R^{*VMI}}{\partial \alpha_{ji}} \geq 0$.

(c) if $(p_i - w_i) \le (p_j - w_j)$, and $-[(p_i - w_i) - (p_j - w_j)] \frac{\partial S_i}{\partial q_i} \ge s_i \frac{\partial Sh_i}{\partial q_i}$, then the retailer's profits are (weakly) decreasing in α_{ji}, *i.e.,* $\frac{\partial \pi_R^{*VMI}}{\partial \alpha_{ji}} \le 0$.

Proposition 2 (a) addresses the case when substitution from one product to another changes, but not vice versa. Proposition 2 (ii) (b) and (ii) (c) show that this may or may not benefit the retailer. For example, if $(p_i - w_i) << (p_j - w_j)$, then an increase α_{ji} is likely to hurt the retailer.

4.3 RMI for one product, VMI for the other

Proposition ***3***

For a given w_i, w_j, p_i, p_j, and full backorders, under RMI for product *i*, and VMI for product *j*, $i \in \{1,2\}, i \ne j$,

(*i*) an increase in α_{ij},will make $q_i^{*RMI_iVMI_j}$ (weakly) decrease but $q_j^{*RMI_iVMI_j}$ increase (weakly),

(ii) an increase in α_{ji}, will make $q_i^{*RMI_iVMI_j}$ (weakly) increase but $q_j^{*RMI_iVMI_j}$ decrease (weakly),

(iii) (a) if, $(p_i - w_i) \le (p_j - w_j)$, $-\{[(p_i - w_i) - (p_j - w_j)](\frac{\partial S_i}{\partial q_i}) - h_i(\frac{\partial I_i}{\partial q_i})\} \ge s_i(\frac{\partial Sh_i}{\partial q_i})$,

and

$$[-(p_i - w_i) + (p_j - w_j)](\frac{\partial S_j}{\partial q_j}) + s_j(\frac{\partial Sh_j}{\partial q_j}) \ge h_i(\frac{\partial I_i}{\partial q_j})$$

then $\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial \alpha_{ij}} \ge 0$, and $\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial \alpha_{ji}} \le 0$.

(b) if, $(p_i - w_i) \ge (p_j - w_j)$

$$[(p_i - w_i) - (p_j - w_j)](\frac{\partial S_i}{\partial q_i}) + s_i(\frac{\partial Sh_i}{\partial q_i}) \ge h_i(\frac{\partial I_i}{\partial q_i}),$$

and

$$-\{[-(p_i-w_i)+(p_j-w_j)](\frac{\partial S_j}{\partial q_j})-h_i(\frac{\partial I_i}{\partial q_j})\}\geq s_j(\frac{\partial Sh_j}{\partial q_j})$$

then $\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial \alpha_{ij}} \leq 0$, and $\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial \alpha_{ji}} \geq 0$.

Propositions 3 (*i*) and (ii) state that a change in substitution from one product to another will make the stock of one product grow but the other go down, just as in Proposition 2 (ii). Propositions 3 (iii) (a) and (b) state that, whether this change in stocking quantities benefits the retailer depends on the retailer's margins, holding cost, and goodwill loss for each product. Thus, under this scenario, a change in substitution may benefit or hurt the retailer. Just as in our example in section 4.3, if $(p_i-w_i)<<(p_j-w_j)$, and increase in α_{ij}, is likely to benefit the retailer, while an increase in α_{ji} is likely to hurt him.

4.4 Comparisons between the three cases

Proposition *4*

For a given w_i, w_j, p_i, p_j, and full backorders, $i \in \{1,2\}, i \neq j$,

(i) if $\alpha_{ij}=\alpha_{ji}=\alpha=0$, or if d_i, d_j, are perfectly positively correlated, then the stocking levels are identical under RMI, VMI, and RMI_iVMI_j.

(ii) If everything is symmetric (*i.e.* both products are identical), $(w-c)$ is small enough, and s small enough, then:

(a) $\exists\, s'$, $(w-c)'$ such that, for $s''<s'$, $(w-c)''<(w-c)'$ implies $q_i^{*RMI} \geq q_i^{*VMI}$ $i \in \{1,2\}$, with similar results for asymmetric cases, and

(b) $q_j^{*RMI} \geq q_j^{*RMI_iVMI_j}$, with similar results for asymmetric cases.

(iii) If everything is symmetric (*i.e.* both products are identical), and $(w-c)$ is large enough, then:

(a) $q^{*RMI} \leq q^{*VMI}$, with similar results for asymmetric cases, and

(b) $q_j^{*RMI} \leq q_j^{*RMI_iVMI_j}$ with similar results for asymmetric cases.

(iv) For asymmetric cases, for $(p_i - w_i)$ large enough, and s_j small enough,

(a) $q_j^{*RMI} \le q_j^{*VMI}$ but $q_i^{*RMI} \ge q_i^{*VMI}$, and

(b) $q_j^{*RMI} \le q_j^{*RMI_iVMI_j}$ but $q_i^{*RMI} \ge q_i^{*RMI_iVMI_j}$.

(v) If $(p_j - w_j)$ large enough, $q_j^{*RMI} \ge q_j^{*RMI_iVMI_j}$.

Proposition 4 (*i*) is a direct consequence of the backorder assumption. Absent substitution, no orders would ever be lost for each manufacturer, regardless of whether the competing product stocked out or not, or the quantity of its own product stocked, and given this, the retailer's first order conditions for RMI, each manufacturers first order condition under VMI, and the retailer's and manufacturer's first order conditions for RMI_iVMI_j are identical, because the quantity stocked is only influencing inventory and sales. This result is exactly as in Mishra and Raghunathan (2004). As we will see later, this need no longer be true when sales are lost.

Propositions 4 (ii) (a), (b), (iii) (a) and (b), and (iv) (a) and (b) highlight the fact that the quantities stocked under VMI are heavily dependent on each *manufacturer's* margins, while, under RMI, it is the *retailer's* margins that matter. Thus, if a manufacturer's margins are very small compared to the retailer's, she will stock less under VMI than under RMI, and, similarly, if her margins are large compared to the retailer, she will stock more under VMI than under RMI. As it will be shown later, this result will still hold when lost sales, rather than backorders, are assumed. Proposition 5 will explore how the retailer can exploit this effect in different scenarios.

Proposition *5*

For a given w_i, w_j, p_i, p_j, full backorders, $i \in \{1,2\}, i \neq j$, if

(*i*)

(a) $(w_i - c_i)$ and $(w_j - c_j)$ are large enough, and $(p_j - w_j) \approx (p_i - w_i)$, or

(b) $\alpha_{ij} = \alpha_{ji} = \alpha = 0$,

then $\pi_R^{*RMI} \le \pi_R^{*VMI}$ and $\pi_R^{*RMI_iVMI_j} \le \pi_R^{*VMI}$.

(ii) $(p_j - w_j)$ and $(p_i - w_i)$ are large enough, $(w_i - c_i)$, s_i, $(w_j - c_j)$, and s_j are small enough, then $\pi_R^{*RMI} \ge \pi_R^{*VMI}$ and $\pi_R^{*RMI} \ge \pi_R^{*RMI_iVMI_j}$.

(iii)

(a) $(p_i - w_i)$, $(w_i - c_i)$, $(w_j - c_j)$ are large enough, h_i, h_j and $(p_j - w_j)$ s_j are small enough, and $(p_i - w_i) \le (w_i - c_i)$, or

(b) $(w_i - c_i)$, h_i, s_i are small enough, $(w_j - c_j)$ is large enough, and $(p_i - w_i) = (p_j - w_j)$,

then , $\pi_R^{*RMI_iVMI_j} \ge \pi_R^{*VMI}$ and $\pi_R^{*RMI_iVMI_j} \ge \pi_R^{*RMI}$.

Proposition 4 (iii) leads directly to Proposition 5 (*i*) (a) for, if the manufacturer's margins are large enough, the fact that both quantities go up under VMI with respect to RMI benefits the retailer, given that he does not pay any holding cost when doing VMI on a product. Similarly, Proposition 5 (*i*) (b) follows from Proposition 4 (*i*), which states that, absent substitution, all the quantities are equal in the three cases, and thus VMI maximizes profits for the retailer because he can then shift holding costs to the manufacturers: thus, under full backorders, absent substitution, regardless of margins of either party, the retailer is always better off doing VMI for both products. As we will see later, this is no longer true once lost sales, rather than backorders, are assumed.

Proposition 5 (ii) follows similar logic: if the manufacturer's margins are low enough, then, as Proposition 4 (ii) shows, moving to VMI either product would make the manufacturer stock less than under RMI. If the retailer's margins are large enough, then the retailer would be at least as well off doing RMI for both products, because it would pay for him to take up the holding cost of stocking both quantities in exchange of the larger sales.

Proposition 5 (iii) (a) works the following way: if product *i* is very lucrative, product *j* is not lucrative at all for the retailer, and the cost of underserving the market for product *j* is low enough, then, under RMI, the retailer will stock very little of *j*, essentially diverting all possible demand to the more lucrative product *i*. Moving both products to VMI may hurt the retailer because the manufacturer of product *j* may stock *too much* of product *j,* thus limiting the diversion to the more lucrative (for the retailer) product *i*. Finally, if manufacturer *i* makes even more margins than the retailer on product *i,* moving product *i* to VMI will ensure that she stocks even more than the retailer, but keeping control of product *j* allows the retailer to continue understocking *j* to divert demand to *i*. On top of this, the retailer saves the holding cost on product *i,* which is precisely the product whose stock is the largest.

In Proposition 5 (iii) (b), moving both products to VMI will make manufacturer *i* stock very little, and manufacturer *j* stock a lot. The ideal situation if for the retailer to keep control of product *i* because his margins are larger than manufacturer *i*, but let manufacturer *j* make decisions on product *j* because her margins are larger.

Therefore, a retailer considering whether to ask for VMI from its manufacturers must be fully aware of the intricacies of its situation, its margins vs the manufacturers' margins, and the relative product's margins as well. The combination of Propositions 4 and 5 fully justify our comments in the paper's introduction about how, under full backorders, some asymmetric cases may result in retailers losing money if switching from RMI to VMI. The next section will examine the case of lost sales.

5. Analysis of the Lost Sales Case

As mentioned earlier, for this section, we will assume lost sales instead of backorders, and $s_{ri} = s_{mi} = t_{ij} = 0 \; i \in \{1,2\}, i \neq j$, which makes the mathematics of the lost sales case more tractable. Results can be extended to the more general case, but the general intuition does not change.

5.1 RMI for both products

Proposition *6*

For a given w_i, w_j, p_i, p_j, and lost sales, under RMI, $i \in \{1,2\}, i \neq j$ the retailer's profits are (weakly) increasing in α_{ij}, *i.e.,* $\frac{\partial \pi_R^{*RMI}}{\partial \alpha_{ij}} \geq 0$.

Proposition 6 is compatible with, Noonan (1995), who, in page 8-5, finds through a numerical study that, for symmetric cases and a bivariate normal demand distribution, $\frac{\partial \pi_R^{*RMI}}{\partial \alpha} \geq 0$.

Notice how the conditions for the profits to be increasing in α_{ij} are weaker than in the full backorder case: while in the former it had to be true that the marginal extra sale of product *j*, and the subsequent saving of one shortage of *i*, made up for the lost sale of product *i* (remembering that in the full backorder scenario, demand not transferred to *j* is backordered), in the latter case this is not necessary because the demand that would not switch from product *i* to product *j* is lost[iii].

However, the influence of α on total profit seems highly dependent on the correlation between the two "originating demands" d_i and d_j, $i \in \{1,2\}, i \neq j$: the more correlated the variables, the lower the influence of α in retailer profits. This is consistent with Netessine and Rudi (2003) who show that, for normally distributed demand, retailer profits are decreasing in the coefficient of correlation (although a formal proof will not be given, we expect similar results for the full backorder case).

5.2 VMI for both products

Proposition *7*

For a given $w_i, w_j,\ p_i, p_j, i \in \{1,2\},\ i \neq j$, and lost sales,

(i) an increase in α_{ji} will make q_i^{*VMI} (weakly) increase but q_j^{*VMI} decrease (weakly), and

(ii) if $(p_j - w_j)\alpha_{ij} Pr(d_i > q_i) \leq (p_i - w_i) Pr(d_i^S > q_i) + (p_j - w_j)\alpha_{ij} Pr(d_j^S > q_j, d_i > q_i)$, and $(p_i - w_i)\alpha_{ji} Pr(d_j > q_j) \geq (p_j - w_j) Pr(d_j^S > q_j) + (p_i - w_i)\alpha_{ji} Pr(d_i^S > q_i, d_j > q_j)$, then $\frac{\partial \pi_R^{VMI}}{\partial \alpha_{ji}} \geq 0$. If the inequalities above are reversed, then $\frac{\partial \pi_R^{VMI}}{\partial \alpha_{ji}} \leq 0$.

Proposition 7 (*i*) is the equivalent to Proposition 2 (ii) (a) for the lost sales case. If α_{ji} increases, product *i* sees more demand, thus making manufacturer *i* increase its stocking quantity. An increase in the quantity stocked of product *i* causes product *j* to see less indirect demand, thus making manufacturer *j* lower its stocking quantity.

Proposition 7 (ii) is the equivalent to Proposition 2 (ii) (b) for the lost sales case: the changes in stocking quantities because of α_{ji} may hurt or benefit the retailer: for example, if $(p_i - w_i)$ is much larger than $(p_j - w_j)$ then the increased substitution would benefit the retailer, while if the opposite is true, the retailer will be hurt by an increase in substitution. This is one of the fundamental differences between RMI and VMI for the lost sales case: while, under RMI and lost sales, an increase in α_{ji} resulted in increased optimal profits for a large range of cases, this will no longer be true under VMI and lost sales: only increases in α_{ji} that lead to manufacturers increasing (decreasing) "the right products" make the retailer better off.

5.3 RMI for one product, VMI for the other

Proposition 8

For a given $w_i, w_j,\ p_i, p_j$, and full backorders, under RMI for product *i*, and VMI for product *j*, $i \in \{1,2\}, i \neq j$,

(*i*) an increase in α_{ij} ,will make $q_i^{*RMI_iVMI_j}$ (weakly) decrease but $q_j^{*RMI_iVMI_j}$ increase (weakly),

(ii) an increase in α_{ji}, will make $q_i^{*RMI_iVMI_j}$ (weakly) increase but $q_j^{*RMI_iVMI_j}$ decrease (weakly),

(iii) (a) if

$$(p_j - w_j)\alpha_{ij} Pr(d_i > q_i) - (p_j - w_j)\alpha_{ij} Pr(d_j^S > q_j, d_i > q_i) + (w_i + h_i) Pr(d_i^S < q_i)$$
$$\geq (p_i - w_i) Pr(d_i^S > q_i)$$

and

$$(p_i - w_i)\alpha_{ji} Pr(d_j > q_j) - (p_i - w_i)\alpha_{ji} Pr(d_i^S > q_i, d_j > q_j) \leq (p_j - w_j) Pr(d_j^S > q_j)$$

then $\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial \alpha_{ij}} \geq 0$, and $\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial \alpha_{ji}} \leq 0$

(b) if

$$(p_j - w_j)\alpha_{ij} Pr(d_i > q_i) - (p_j - w_j)\alpha_{ij} Pr(d_j^S > q_j, d_i > q_i) + (w_i + h_i) Pr(d_i^S < q_i)$$
$$\leq (p_i - w_i) Pr(d_i^S > q_i)$$

and

$$(p_i - w_i)\alpha_{ji} Pr(d_j > q_j) - (p_i - w_i)\alpha_{ji} Pr(d_i^S > q_i, d_j > q_j) \geq (p_j - w_j) Pr(d_j^S > q_j)$$

then $\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial \alpha_{ij}} \leq 0$, and $\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial \alpha_{ji}} \geq 0$.

Proposition 8 is the lost sales analogous of Proposition 3. Just as in the backorder case, Proposition 8 (*i*) and (ii) state that a change in substitution from one product to another will make the stock of one product grow but the other go down. Propositions 8 (iii) (a) and (b), just as in the backorders case, state that, whether this change in stocking quantities benefits the retailer depends on the retailer's margins, holding cost of product *i*, and goodwill loss for each product. Thus, under this scenario, a change in substitution may benefit or hurt the retailer. The example given for the backorders case still applies: if $(p_i - w_i) << (p_j - w_j)$, and increase in α_{ij}, is likely to benefit the retailer, while an increase in α_{ji} is likely to hurt him.

5.4 Comparisons between the three cases

Proposition *9*

For a given w_i, w_j, p_i, p_j, x_i x_j not perfectly positively correlated, $i \in \{1,2\}$, $i \neq j$ and lost sales,

(i) The result from Proposition 3 (*i*) needs not hold in the lost sales case, regardless of symmetry. In fact, even for symmetric product demands, and $\alpha_{ij} = \alpha_{ji} = \alpha = 0$, $q_i^{*RMI} \neq q_i^{*VMI}, i \in \{1,2\}$, unless $\frac{(w_i - c_i)}{(w_i + h_i)} = \frac{(p_i - w_i)}{(p_i + h_i)}$.

(ii) If everything is symmetric, demands are iid, and $\frac{(w_i - c_i)}{(w_i + h_i)} = \frac{(p_i - w_i)}{(p_i + h_i)}$, then $q_i^{*RMI} \leq q_i^{*VMI}\ \forall i$.

(iii) If everything is symmetric, and $(w - c)$ is small enough,

(a) $q^{*RMI} \geq q^{*VMI}$, with similar results for asymmetric cases

(b) $q_j^{*RMI} \geq q_j^{*RMI_i VMI_j}$, with similar results for asymmetric cases.

(iv) If everything is symmetric, and $(p - w)$ is large enough, $q^{*RMI} \geq q^{*VMI}$, with similar results for asymmetric cases

(v) If $(p_j - w_j)$ large enough, $q_j^{*RMI} \geq q_j^{*RMI_i VMI_j}$.

(vi) If $(p_i - w_i)$, large enough,

(a) $q_j^{*RMI} \leq q_j^{*VMI}$ but $q_i^{*RMI} \geq q_i^{*VMI}$, and

(b) $q_j^{*RMI} \leq q_j^{*RMI_i VMI_j}$ but $q_i^{*RMI} \geq q_i^{*RMI_i VMI_j}$.

(vii) For symmetric cases, for $(p - w)$, small enough, or $(w - c)$ large enough,

(a) $q^{*RMI} \leq q^{*VMI}$, with similar results for asymmetric cases, and

(b) $q_j^{*RMI} \leq q_j^{*RMI_i VMI_j}$, with similar results for asymmetric cases.

Proposition 9 (*i*) is a consequence of the fact that, absent substitution, both VMI and RMI turn into standard newsvendor problems. However, under VMI, the understocking cost is $(w_i - c_i)$, while under RMI is $(p_i - w_i)$.

Proposition 9 (ii) was noted by Netessine and Rudi (2003): in this particular case, manufacturers under VMI stock more than under RMI because they do not care for sales of the other product through substitution, and thus the (potentially) negative effect on the retailer's profits of an increase in some manufacturer's stocking levels is ignored by the manufacturer.

Propositions 9 (iii), (iv) and (v), (vi) and (vii) are the lost sales analogous to Proposition 4 (ii), (iii), (iv) and (v).

Proposition *10*

For a given w_i, w_j, p_i, p_j, and lost sales, $i \in \{1,2\}, i \neq j$, if

(*i*) $(w_i - c_i)$ and $(w_j - c_j)$ are large enough, and either

(a) $(p_j - w_j) \approx (p_i - w_i)$, or

(b) $\alpha_{ij} = \alpha_{ji} = \alpha = 0$,

then $\pi_R^{*RMI} \leq \pi_R^{*VMI}$ and $\pi_R^{*RMI_iVMI_j} \leq \pi_R^{*VMI}$.

(ii) $(p_j - w_j)$ and $(p_i - w_i)$ are large enough, $(w_i - c_i)$, and $(w_j - c_j)$, are small enough, then $\pi_R^{*RMI} \geq \pi_R^{*VMI}$ and $\pi_R^{*RMI} \geq \pi_R^{*RMI_iVMI_j}$.

(iii)

(a) $(p_i - w_i)$, $(w_i - c_i)$, $(w_j - c_j)$ are large enough, h_i, h_j and $(p_j - w_j)$ are small enough, and $(p_i - w_i) \leq (w_i - c_i)$, then $\pi_R^{*RMI_jVMI_i} \geq \pi_R^{*VMI}$ and $\pi_R^{*RMI_jVMI_i} \geq \pi_R^{*RMI}$.

(b) $(w_i - c_i)$, an h_i are small enough, $(w_j - c_j)$ is large enough, and $(p_i - w_i) = (p_j - w_j)$, then , $\pi_R^{*RMI_iVMI_j} \geq \pi_R^{*VMI}$ and $\pi_R^{*RMI_iVMI_j} \geq \pi_R^{*RMI}$.

Proposition 10 follows the exact logic of Proposition 5, generalizing those results to the lost sales case. The one difference is that, while under full backorders, absent substitution the quantities stocked under VMI, RMI and RMI_iVMI_j where all the same, this is no longer true when lost sales are assumed. Because of this, in cases where there is no substitution, if retailer margins are significantly lower (higher) than either manufacturer's margins, the retailer is better off giving stocking decision rights to the manufacturer (retailer): *i.e.* if either party has significantly higher margins, he or she should have decision rights. In the backorder case, the rule was, "absent substitution, always do VMI for both products".

6. Discussion and Conclusions

Analyzing a two-product full substitution case, this paper qualifies two beliefs stated in the literature:

(1) that, under VMI, the retailers would benefit because manufacturers would increase stocking quantities as a result of competition, and

(2) that substitution benefits retailers.

We find that idea (1), while appealing, simply is not true for many cases, specially when manufacturer's margins are significantly less than those of retailers', or when two products are highly asymmetric. Moreover, we find that as a consequence of this belief not being true, it is perfectly possible for VMI to hurt retailers instead of benefiting them. Our findings apply both to full backorders and lost sales case.

We also find that idea (2) does not hold for a wide number of cases: for full backorders, this will not be true both in RMI or VMI if the substitution that increases is from the most lucrative to the least lucrative product: if this is the case, customers would simply be "trading down"; for lost sales, we find that a similar situation arises under VMI but not under RMI. In both the full backorders and lost sales cases, if one product is under RMI and the other under VMI, substitution may also hurt or benefit the retailer, depending on relative margins, holding costs, and stockout costs.

Propositions 5 and 10 have managerial implications: they provide insights for retailers to know when to do RMI for both products, when to do RMI for one product and VMI for another, and when to do VMI for both products. The model in this paper has, of course, its limitations: (a) it does not account for the fact that the retailer and the manufacturer's may have different capabilities, (b) does not consider the fact that manufacturers may be able to have a clearer overall picture of demand for its product because they see several retailers, and (c) limits the analysis to exogenously determined, price-only contracts. Although this paper constitutes one attempt to further our understanding of the effects on retailer profits of competition between substitute products and vendor managed inventory, there are still numerous opportunities for further research on either topic.

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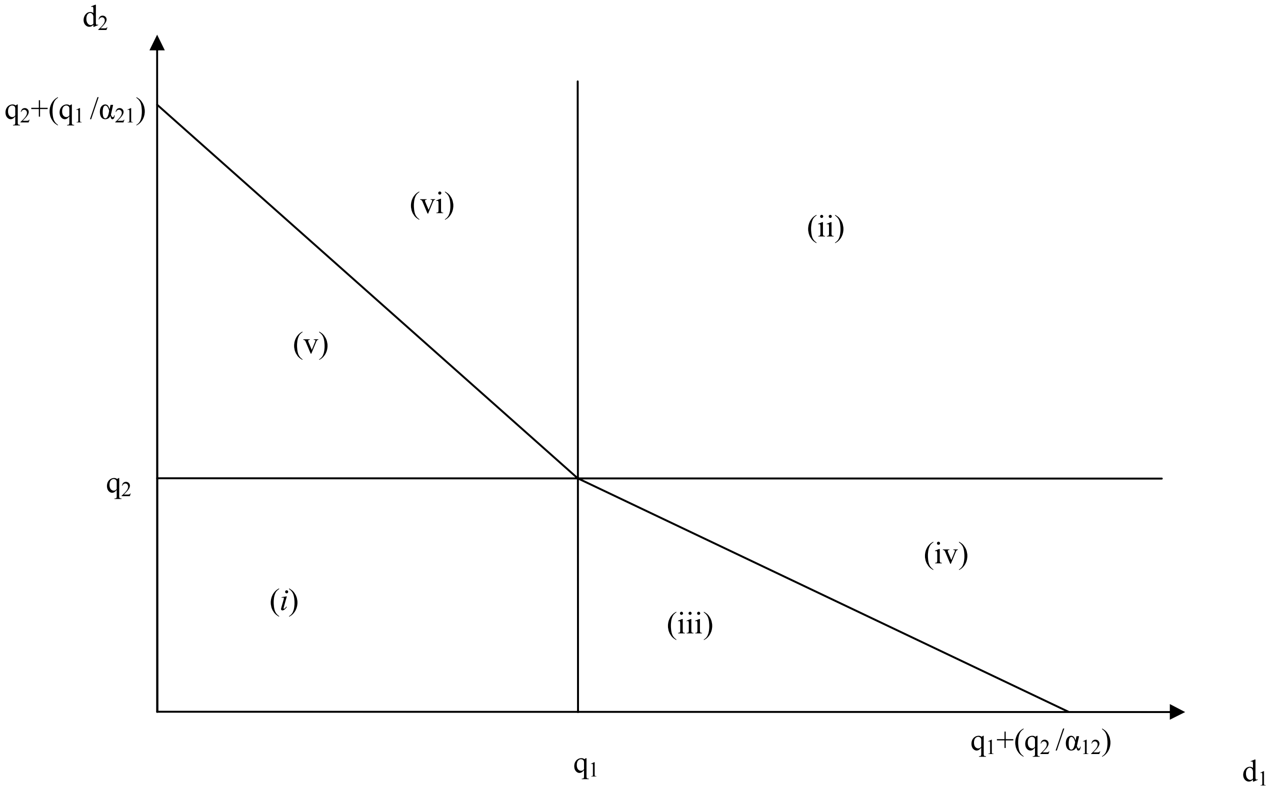
Appendix 1: Figures

Figure 1: Demand structure -Graph as in Noonan (1995)-



Note that the slopes of the diagonal lines need not be equal, as they are determined by α_{ij}.

Figure 2: RMI for both products

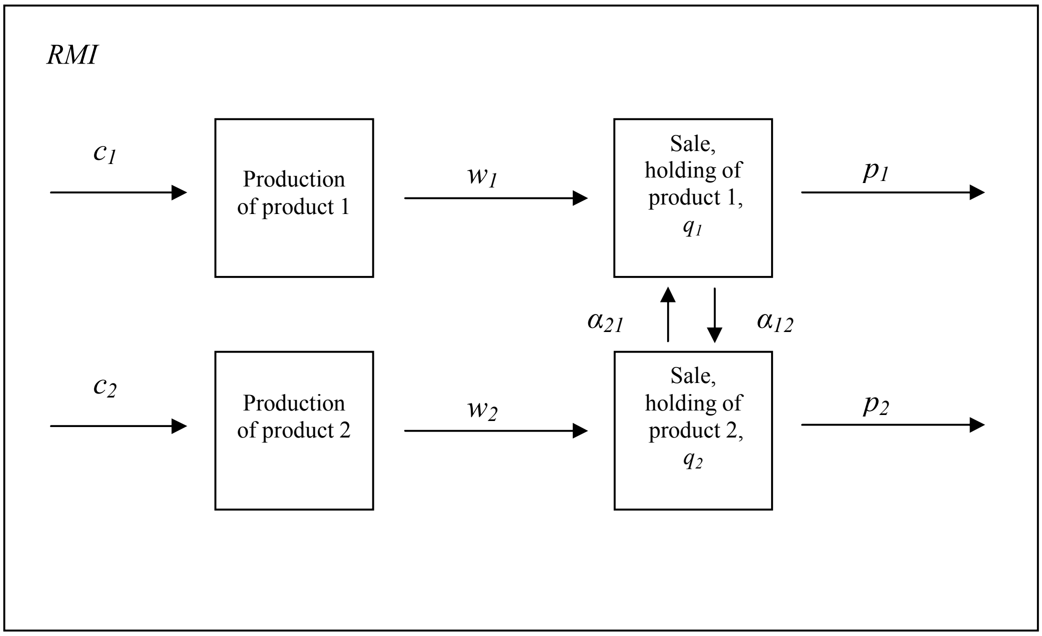


Figure 3: VMI for both products.

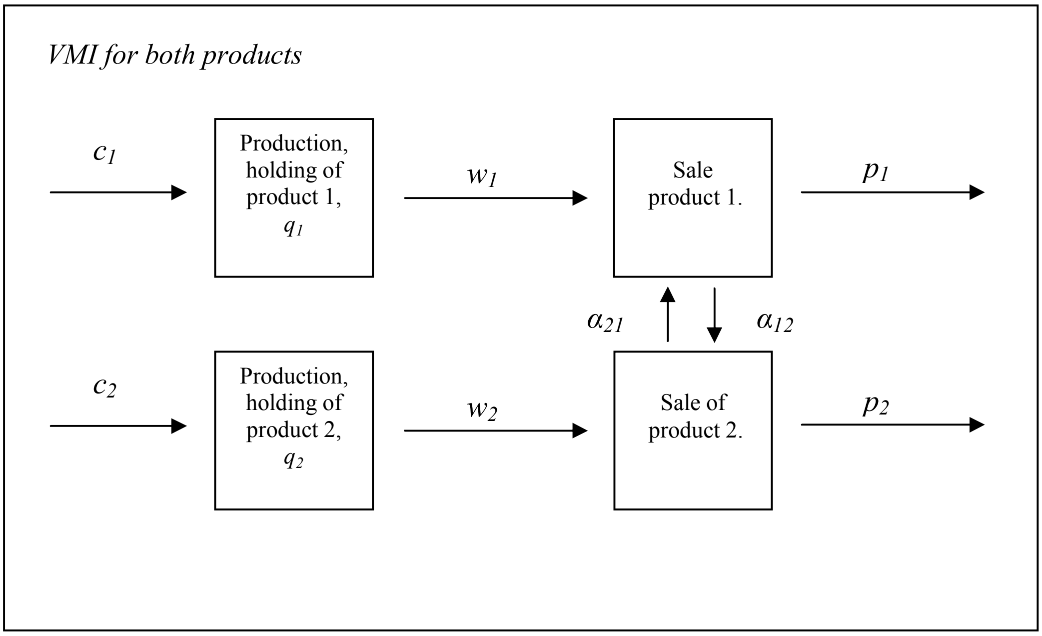
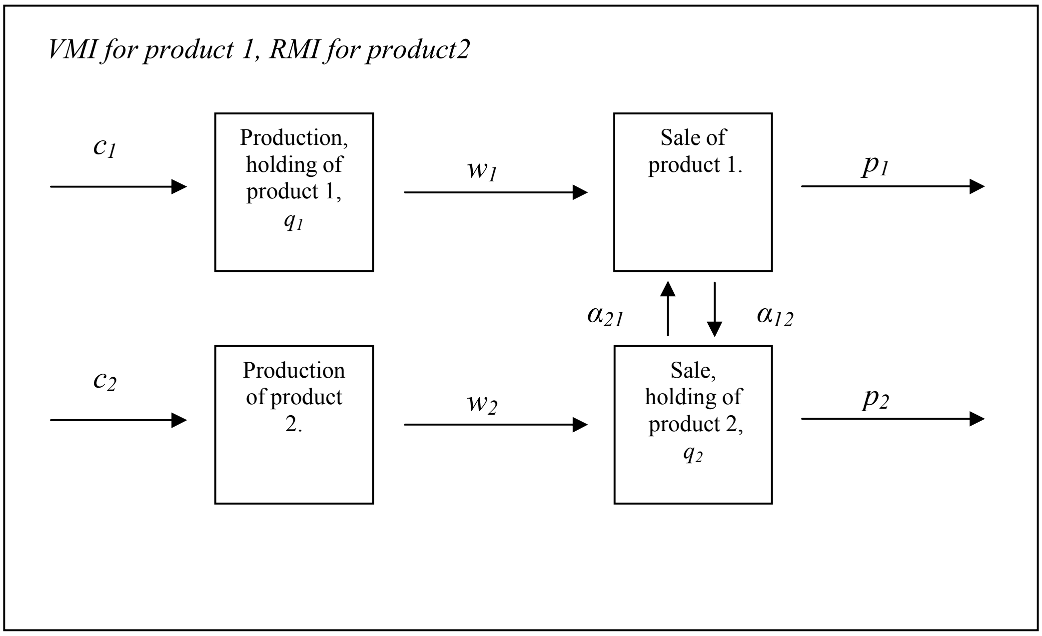


Figure 4: VMI for one product, RMI for another.



Appendix 2: Proofs

Throughout the proofs, π_R^{RMI} will denote the retailer's profits under RMI, π_R^{VMI}, the retailer's profits under VMI for both products, and $\pi_R^{RMI_i VMI_j}$ the retailer's profits when doing RMI with product *i*, and VMI with product *j*. Similarly, π_i^{RMI} denotes manufacturer *i*'s profits under RMI, etc.

Proof of Proposition *1*

$$\frac{\partial \pi_R^{RMI}}{\partial \alpha_{ij}} = [-(p_i - w_i) + (p_j - w_j) + (h_j)] \int_0^{q_j} \int_{q_i}^{q_1 + \frac{q_2 - x_2}{\alpha_{12}}} (x_i - q_i) f[x_i] f[x_j] dx_i dx_j$$

Therefore,

(i) if $(p_i - w_i) \le (p_j - w_j) + h_j$, then $\frac{\partial \pi_R^{RMI}}{\partial \alpha_{ij}} \ge 0$,

(ii) if $(p_i - w_i) \ge (p_j - w_j) + h_j$, then $\frac{\partial \pi_R^{RMI}}{\partial \alpha_{ij}} \le 0$.

This is true $\forall$ q_i^{RMI} and q_j^{RMI} .

Proof of Proposition *2*

(a)

The first order conditions for VMI are:

$$\frac{\partial \pi_i^{VMI}}{\partial q_i} = G_{i_{qi}}^{VMI} = (w_i - c_i)(\frac{\partial S_i}{\partial q_i}) - (h_i)(\frac{\partial I_i}{\partial q_i}) - s_i(\frac{\partial Sh_i}{\partial q_i})$$

Then,

$$\frac{\partial^2 \pi_1^{VMI}}{\partial \alpha_{21} \partial q_1} = (w_1 - c_1)\frac{\partial^2 S_1}{\partial \alpha_{21} \partial q_1} - (h_1)(\frac{\partial^2 I_1}{\partial \alpha_{21} \partial q_1}) - s_1(\frac{\partial^2 Sh_1}{\partial \alpha_{21} \partial q_1}) \ge 0$$

because $\frac{\partial^2 S_1}{\partial \alpha_{21} \partial q_1} \ge 0$, $\frac{\partial^2 I_1}{\partial \alpha_{21} \partial q_1} \le 0$, and $\frac{\partial^2 Sh_1}{\partial \alpha_{21} \partial q_1} = 0$

Also

$$\frac{\partial^2 \pi_2^{VMI}}{\partial \alpha_{21} \partial q_2} = (w_2 - c_2)\frac{\partial^2 S_2}{\partial \alpha_{21} \partial q_2} - (h_2)(\frac{\partial^2 I_2}{\partial \alpha_{21} \partial q_2}) - s_2(\frac{\partial^2 Sh_2}{\partial \alpha_{21} \partial q_2}) = 0$$

because $\frac{\partial^2 S_2}{\partial \alpha_{21} \partial q_2} = 0$, $\frac{\partial^2 I_1}{\partial \alpha_{21} \partial q_1} = 0$, and $\frac{\partial^2 Sh_1}{\partial \alpha_{21} \partial q_1} = 0$

So, the direct effect of an increase in α_{21} is an increase in q_1^{*VMI}, and there is no direct effect on q_2^{*VMI}. However, this does not mean that q_2^{*VMI} will not change, since:

$$\frac{\partial^2 \pi_2^{VMI}}{\partial q_1 \partial q_2} = (w_2 - c_2)\frac{\partial^2 S_2}{\partial q_1 \partial q_2} - h_2(\frac{\partial^2 I_2}{\partial q_1 \partial q_2}) - s_2(\frac{\partial^2 Sh_2}{\partial q_1 \partial q_2}) \leq 0$$

because $\frac{\partial^2 S_2}{\partial q_2 \partial q_1} \leq 0$, $\frac{\partial^2 I_2}{\partial q_2 \partial q_1} \geq 0$, and $\frac{\partial^2 Sh_2}{\partial q_2 \partial q_1} = 0$

Therefore, the direct effect of an increase in q_1^{*VMI} is a decrease in q_2^{*VMI}.

Finally, using the same logic,

$$\frac{\partial^2 \pi_1^{VMI}}{\partial q_2 \partial q_1} \leq 0$$

So, a decrease in q_2^{*VMI} increases q_1^{*VMI}.

This completes the proof. Note that proof is without loss of generality, and would therefore apply to an increase in α_{12}.

(b) and (c)

We know that

$$\frac{\partial \pi_R^{VMI}}{\partial q_1} = [(p_1 - w_1) - (p_2 - w_2)](\frac{\partial S_1}{\partial q_1}) - s_1(\frac{\partial Sh_1}{\partial q_1})$$

and

$$\frac{\partial\ \pi_R^{VMI}}{\partial q_2} = [(p_2 - w_2) - (p_1 - w_1)](\frac{\partial\ S_2}{\partial q_2}) - s_2(\frac{\partial\ Sh_2}{\partial q_2})$$

Therefore,

(a.1) if $(p_1 - w_1) \geq (p_2 - w_2)$, then $\frac{\partial\ \pi_R^{VMI}}{\partial q_1} \geq 0$

(a.2) if $(p_1 - w_1) \geq (p_2 - w_2)$, and $[(p_2 - w_2) - (p_1 - w_1)](\frac{\partial\ S_2}{\partial q_2}) \leq s_2(\frac{\partial\ Sh_2}{\partial q_2})$, then $\frac{\partial\ \pi_R^{VMI}}{\partial q_2} \leq 0$

(a.3) $(p_1 - w_1) \leq (p_2 - w_2)$, then $\frac{\partial\ \pi_R^{VMI}}{\partial q_2} \geq 0$

(a.4) if $(p_1 - w_1) \leq (p_2 - w_2)$, and $-[(p_1 - w_1) - (p_2 - w_2)](\frac{\partial\ S_1}{\partial q_1}) \geq s_1(\frac{\partial\ Sh_1}{\partial q_1})$, then $\frac{\partial\ \pi_R^{VMI}}{\partial q_1} \leq 0$.

Combining (a.1) to (a.4) with the result from Proposition 2 (a) gets the desired result.

Proof of Proposition *3*

(*i*)

Under RMI_iVMI_j, the first order conditions for the manufacturer doing VMI are:

$$\frac{\partial\ \pi_j}{\partial q_j} = G_{j_{qj}}^{RMI_iVMI_j} = (w_j - c_j)(\frac{\partial\ S_j}{\partial q_j}) - (h_j)(\frac{\partial\ I_j}{\partial q_j}) - s_j(\frac{\partial\ Sh_j}{\partial q_j})$$

Then,

$$\frac{\partial^2 \pi_j^{RMI_iVMI_j}}{\partial \alpha_{ij} \partial q_j} = (w_j - c_j)\frac{\partial^2 S_j}{\partial \alpha_{ij} \partial q_j} - (h_j)(\frac{\partial^2 I_j}{\partial \alpha_{ij} \partial q_j}) - s_j(\frac{\partial^2 Sh_j}{\partial \alpha_{ij} \partial q_j}) \geq 0$$

because $\frac{\partial^2 S_j}{\partial \alpha_{ij} \partial q_j} \geq 0$, $\frac{\partial^2 I_j}{\partial \alpha_{ij} \partial q_j} \leq 0$, and $\frac{\partial^2 Sh_j}{\partial \alpha_{ij} \partial q_j} = 0$

So, the direct effect of an increase in α_{ij} is an increase in $q_j^{*RMI_iVMI_j}$.

Also,

$$\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial q_i} = G_{R_{qi}}^{RMI_iVMI_j} = (p_i - w_i)(\frac{\partial S_i}{\partial q_i}) + (p_j - w_j)(\frac{\partial S_j}{\partial q_i}) - h_i(\frac{\partial I_i}{\partial q_i})$$
$$- s_i(\frac{\partial Sh_i}{\partial q_i}) - s_j(\frac{\partial Sh_j}{\partial q_i})$$

$$= [(p_i - w_i) - (p_j - w_j)](\frac{\partial S_i}{\partial q_i}) - h_i(\frac{\partial I_i}{\partial q_i}) - s_i(\frac{\partial Sh_i}{\partial q_i})$$

So,

$$\frac{\partial^2 \pi_R^{RMI_iVMI_j}}{\partial \alpha_{ij} \partial q_i} = [(p_i - w_i) - (p_j - w_j)](\frac{\partial^2 S_i}{\partial \alpha_{ij} \partial q_i}) - h_i(\frac{\partial^2 I_i}{\partial \alpha_{ij} \partial q_i}) - s_i(\frac{\partial^2 Sh_i}{\partial \alpha_{ij} \partial q_i}) \leq 0$$

because $\frac{\partial^2 S_i}{\partial \alpha_{ij} \partial q_i} \leq 0$, $\frac{\partial^2 I_i}{\partial \alpha_{ij} \partial q_i} = 0$, and $\frac{\partial^2 Sh_i}{\partial \alpha_{ij} \partial q_i} = 0$.

So, the direct effect of an increase in α_{ij} is an increase in $q_j^{*RMI_iVMI_j}$, and a decrease on $q_i^{*RMI_iVMI_j}$.
Next:

$$\frac{\partial^2 \pi_R^{RMI_iVMI_j}}{\partial q_j \partial q_i} = [(p_i - w_i) - (p_j - w_j)] \frac{\partial^2 S_i}{\partial q_j \partial q_i} - h_i(\frac{\partial^2 I_i}{\partial q_j \partial q_i}) - s_i(\frac{\partial^2 Sh_i}{\partial q_j \partial q_i}) \leq 0$$

because $\frac{\partial^2 S_i}{\partial q_j \partial q_i} \leq 0$, $\frac{\partial^2 I_i}{\partial q_j \partial q_i} \geq 0$, and $\frac{\partial^2 Sh_i}{\partial q_j \partial q_i} = 0$.

Therefore, the direct effect of an increase in $q_j^{*RMI_iVMI_j}$ is a decrease in $q_i^{*RMI_iVMI_j}$.

Finally, using the same logic,

$$\frac{\partial^2 \pi_j^{RMI_iVMI_j}}{\partial q_i \partial q_j} = (w_j - c_j)\frac{\partial^2 S_j}{\partial \alpha_{ij} \partial q_j} - (h_j)(\frac{\partial^2 I_j}{\partial \alpha_{ij} \partial q_j}) - s_j(\frac{\partial^2 Sh_j}{\partial \alpha_{ij} \partial q_j}) \leq 0$$

because $\frac{\partial^2 S_j}{\partial q_i \partial q_j} \leq 0$, $\frac{\partial^2 I_j}{\partial q_i \partial q_j} \leq 0$, and $\frac{\partial^2 Sh_j}{\partial q_i \partial q_j} = 0$.

So, a decrease in $q_i^{*RMI_iVMI_j}$ increases $q_j^{*RMI_iVMI_j}$.

This completes the proof.

(ii)

$$\frac{\partial^2 \pi_R^{RMI_iVMI_j}}{\partial \alpha_{ji} \partial q_i} = [(p_i - w_i) - (p_j - w_j)](\frac{\partial^2 S_i}{\partial \alpha_{ji} \partial q_i}) - h_i(\frac{\partial^2 I_i}{\partial \alpha_{ji} \partial q_i}) - s_i(\frac{\partial^2 Sh_i}{\partial \alpha_{ji} \partial q_i}) \geq 0$$

because $\frac{\partial^2 S_i}{\partial \alpha_{ji} \partial q_i} \geq 0$, $\frac{\partial^2 I_i}{\partial \alpha_{ij} \partial q_i} \leq 0$, and $\frac{\partial^2 Sh_i}{\partial \alpha_{ij} \partial q_i} = 0$.

So an increase in α_{ji} will make $q_i^{*RMI_iVMI_j}$ increase.

$$\frac{\partial^2 \pi_j^{RMI_iVMI_j}}{\partial \alpha_{ji} \partial q_j} = (w_j - c_j)\frac{\partial^2 S_j}{\partial \alpha_{ji} \partial q_j} - (h_j)(\frac{\partial^2 I_j}{\partial \alpha_{ji} \partial q_j}) - s_j(\frac{\partial^2 Sh_j}{\partial \alpha_{ji} \partial q_j}) \leq 0$$

because $\frac{\partial^2 S_j}{\partial \alpha_{ji} \partial q_j} \leq 0$, $\frac{\partial^2 I_j}{\partial \alpha_{ij} \partial q_j} = 0$, and $\frac{\partial^2 Sh_j}{\partial \alpha_{ij} \partial q_j} = 0$.

So an increase in α_{ji} will make $q_j^{*RMI_iVMI_j}$ decrease. Just as before,

$\frac{\partial^2 \pi_j^{RMI_iVMI_j}}{\partial q_i \partial q_j} \leq 0$, and a decrease in $q_j^{*RMI_iVMI_j}$ causes $q_i^{*RMI_iVMI_j}$ to increase, since

$\frac{\partial^2 \pi_R^{RMI_iVMI_j}}{\partial q_j \partial q_i} \leq 0$.

This completes the proof.

(iii) and (iv)

The effect of a change in α_{ij} on the retailer's profits is through the change in q_j, q_i . As we know from Proposition 3 (*i*) and (ii),

(*i*) an increase in α_{ij}, will make $q_i^{*RMI_iVMI_j}$ (weakly) decrease but $q_j^{*RMI_iVMI_j}$ increase (weakly),

(ii) an increase in α_{ji}, will make $q_i^{*RMI_iVMI_j}$ (weakly) increase but $q_j^{*RMI_iVMI_j}$ decrease (weakly),

How does this affect the retailer's profits? We know that:

$$\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial q_j} = G_{R_{qj}}^{RMI_iVMI_j} = (p_i - w_i)(\frac{\partial S_i}{\partial q_j}) + (p_j - w_j)(\frac{\partial S_j}{\partial q_j}) - h_i(\frac{\partial I_i}{\partial q_j})$$
$$- s_i(\frac{\partial Sh_i}{\partial q_j}) - s_j(\frac{\partial Sh_j}{\partial q_j})$$

$$= [-(p_i - w_i) + (p_j - w_j)](\frac{\partial S_j}{\partial q_j}) - h_i(\frac{\partial I_i}{\partial q_j}) - s_j(\frac{\partial Sh_j}{\partial q_j})$$

and

$$\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial q_i} = G_{R_{qj}}^{RMI_iVMI_j} = (p_i - w_i)(\frac{\partial S_i}{\partial q_i}) + (p_j - w_j)(\frac{\partial S_j}{\partial q_i}) - h_i(\frac{\partial I_i}{\partial q_i})$$
$$- s_i(\frac{\partial Sh_i}{\partial q_i}) - s_j(\frac{\partial Sh_j}{\partial q_i})$$

$$= [(p_i - w_i) - (p_j - w_j)](\frac{\partial S_i}{\partial q_i}) - h_i(\frac{\partial I_i}{\partial q_i}) - s_i(\frac{\partial Sh_i}{\partial q_i})$$

Therefore,

(iii) (a) if, $(p_i - w_i) \le (p_j - w_j)$, $\quad -\{[(p_i - w_i) - (p_j - w_j)](\frac{\partial S_i}{\partial q_i}) - h_i(\frac{\partial I_i}{\partial q_i})\} \ge s_i(\frac{\partial Sh_i}{\partial q_i})$,

and

$$[-(p_i - w_i) + (p_j - w_j)](\frac{\partial S_j}{\partial q_j}) + s_j(\frac{\partial Sh_j}{\partial q_j}) \ge h_i(\frac{\partial I_i}{\partial q_j})$$

then $\dfrac{\partial \pi_R^{RMI_iVMI_j}}{\partial \alpha_{ij}} \geq 0$, and $\dfrac{\partial \pi_R^{RMI_iVMI_j}}{\partial \alpha_{ji}} \leq 0$.

(b) if, $(p_i - w_i) \geq (p_j - w_j)$

$$[(p_i - w_i) - (p_j - w_j)](\frac{\partial S_i}{\partial q_i}) + s_i(\frac{\partial Sh_i}{\partial q_i}) \geq h_i(\frac{\partial I_i}{\partial q_i}),$$

and

$$-\{[-(p_i - w_i) + (p_j - w_j)](\frac{\partial S_j}{\partial q_j}) - h_i(\frac{\partial I_i}{\partial q_j})\} \geq s_j(\frac{\partial Sh_j}{\partial q_j})$$

then $\dfrac{\partial \pi_R^{RMI_iVMI_j}}{\partial \alpha_{ij}} \leq 0$, and $\dfrac{\partial \pi_R^{RMI_iVMI_j}}{\partial \alpha_{ji}} \geq 0$.

Poof of Proposition *4*

(i)

Because when $\alpha = 0$, $\dfrac{\partial S_i}{\partial q_i} = 0$ -as Mishra and Raghunathan (2004) point out-, then the first order conditions under RMI, VMI, and RMI_iVMI_j. are equal.

(ii)(a)

In RMI,

$$\frac{\partial^2 \pi_R}{\partial (w-c) \partial q_i} = 0$$

Therefore, nor q_1^{*RMI}, neither q_2^{*RMI} change as $(w-c)$ changes.

However, in VMI,

$$\frac{\partial^2 \pi_i^{VMI}}{\partial (w-c) \partial q_i} = \frac{\partial S_i}{\partial q_i} \geq 0$$

Also, for $s \to 0$,

$$\frac{\partial\ \pi_i^{VMI}}{\partial q_i} = G_{i_{qi}}^{VMI} \rightarrow (w-c)(\frac{\partial\ S_i}{\partial q_i}) - (h)(\frac{\partial\ I_i}{\partial q_i})$$

So, as $s \rightarrow 0$, and $(w-c) \rightarrow 0, q_i^{*VMI} \rightarrow 0, i \in \{1,2\}$.

Therefore, $\exists\ s'$, $(w-c)'$ such that, for $s'' < s'$, $(w-c)'' < (w-c)'$ implies $q_i^{*RMI} \geq q_i^{*VMI}\ i \in \{1,2\}$.

For asymmetric cases, the result would be: $\exists$ $s_i', (w_i - c_i)'$ such that, for $s_i'' < s_i', (w_i - c_i)'' < (w_i - c_i)'$ implies $q_i^{*RMI} \leq q_i^{*VMI}\ i \in \{1,2\}$.

(ii) (b)

Similarly to the (a) case, under RMI,

$$\frac{\partial^2 \pi_R^{RMI}}{\partial (w-c) \partial q_j} = 0,$$

and, for $s \rightarrow 0$, in RMI$_i$VMI$_j$,

$$\frac{\partial\ \pi_j^{RMI_iVMI_j}}{\partial q_j} = G_{i_{qj}}^{RMI_jVMI_i} \rightarrow (w-c)(\frac{\partial\ S_j}{\partial q_j}) - (h)(\frac{\partial\ I_j}{\partial q_j})$$

So, as $s \rightarrow 0$, and $(w-c) \rightarrow 0, q_j^{*RMI_iVMI_j} \rightarrow 0$.

Therefore, $\exists\ s'$, $(w-c)'$ such that, for $s'' < s'$, $(w-c)'' < (w-c)'$ implies $q_j^{*RMI} \geq q_j^{*RMI_iVMI_j}$.

For asymmetric cases, the result would be: $\exists$ $s_j', (w_j - c_j)'$ such that, for $s_j'' < s_j', (w_j - c_j)'' < (w_j - c_j)'$ implies $q_j^{*RMI} \geq q_j^{*RMI_iVMI_j}$.

(iii)(a)

$$\frac{\partial\ \pi_i^{VMI}}{\partial q_i} = G_{i_{qi}}^{VMI} = (w-c)(\frac{\partial\ S_i}{\partial q_i}) - (h)(\frac{\partial\ I_i}{\partial q_i}) - (s)(\frac{\partial\ Sh_i}{\partial q_i})$$

if $(w-c) \rightarrow \infty$, then $q_i^{*VMI} \rightarrow \infty, i \in \{1,2\}$.

But, as Proposition 3 (ii) showed, nor q_1^{*RMI}, neither q_2^{*RMI} change as $(w-c)$ changes.

Thus, $\exists\ (w-c)'$ such that, for $(w-c)'' > (w-c)'$ implies $q_i^{*RMI} \le q_i^{*VMI}\ i \in \{1,2\}$.

For asymmetric cases, the result would be: $\exists\ (w_i - c_i)'$ such that, for $(w_i - c_i)'' > (w_i - c_i)'$ implies $q_i^{*RMI} \le q_i^{*VMI}\ i \in \{1,2\}$.

(iii) (b)

Just as shown in the proof of Proposition 3 (ii) (b),

$$\frac{\partial \pi_j^{RMI_iVMI_j}}{\partial q_j} = G_{i_{qj}}^{RMI_jVMI_i} = (w-c)\left(\frac{\partial S_j}{\partial q_j}\right) - (h)\left(\frac{\partial I_j}{\partial q_j}\right) - (s)\left(\frac{\partial Sh_j}{\partial q_j}\right),$$

so, if $(w-c) \to \infty$, then $q_j^{*RMI_iVMI_j} \to \infty$.

Again, as Proposition 3 (ii) showed, nor q_1^{*RMI}, neither q_2^{*RMI} change as $(w-c)$ changes.

Therefore, $\exists\ (w-c)'$ such that, for, $(w-c)'' > (w-c)'$ implies $q_j^{*RMI} \le q_j^{*RMI_iVMI_j}$.

For asymmetric cases, the result would be: $\exists\ (w_j - c_j)'$ such that, for $(w_j - c_j)'' > (w_j - c_j)'$ implies $q_j^{*RMI} \le q_j^{*RMI_iVMI_j}$.

(iv)(a)

The first order conditions for RMI are given by:

$$\frac{\partial \pi_R^{RMI}}{\partial q_i} = G_{R_{qi}}^{RMI} = (p_i - w_i)\left(\frac{\partial S_i}{\partial q_i}\right) + (p_j - w_j)\left(\frac{\partial S_j}{\partial q_i}\right) - h_i\left(\frac{\partial I_i}{\partial q_i}\right) - h_j\left(\frac{\partial I_j}{\partial q_i}\right)$$
$$- s_i\left(\frac{\partial Sh_i}{\partial q_i}\right) - s_j\left(\frac{\partial Sh_j}{\partial q_i}\right)$$

$$= [(p_i - w_i) - (p_j - w_j)]\left(\frac{\partial S_j}{\partial q_i}\right) - h_i\left(\frac{\partial I_i}{\partial q_i}\right) - h_j\left(\frac{\partial I_j}{\partial q_i}\right)$$
$$- s_i\left(\frac{\partial Sh_i}{\partial q_i}\right)$$

So,

$$\frac{\partial^2 \pi_R^{RMI}}{\partial(p_1 - w_1)\partial q_1} = \frac{\partial\, G_{R_{q1}}^{RMI}}{\partial(p_1 - w_1)} = \frac{\partial\, S_1}{\partial q_1} \geq 0$$

and

$$\frac{\partial^2 \pi_R^{RMI}}{\partial(p_1 - w_1)\partial q_2} = \frac{\partial\, G_{R_{q2}}^{RMI}}{\partial(p_1 - w_1)} = -\frac{\partial\, S_2}{\partial q_2} \leq 0$$

Also, for $s_i \to 0$,

$$\frac{\partial\, \pi_R^{RMI}}{\partial q_i} = G_{R_{qi}}^{RMI} \to [(p_i - w_i) - (p_j - w_j)](\frac{\partial\, S_i}{\partial q_i}) - h_i(\frac{\partial\, I_i}{\partial q_i}) - h_j(\frac{\partial\, I_j}{\partial q_i})$$

So, as $s_2 \to 0$, and $(p_1 - w_1) \to \infty, q_1^{*RMI} \to \infty$ and $q_2^{*RMI} \to 0$, essentially because it pays to divert more and more demand from product 2 into product 1. This is without loss of generality, and similar results apply for $(p_2 - w_2)$.

The first order conditions for VMI are:

$$\frac{\partial\, \pi_i^{VMI}}{\partial q_i} = G_{i_{qi}}^{VMI} = (w_i - c_i)(\frac{\partial\, S_i}{\partial q_i}) - h_i(\frac{\partial\, I_i}{\partial q_i}) - s_i(\frac{\partial\, Sh_i}{\partial q_i})$$

So,

$$\frac{\partial^2 \pi_1}{\partial(p_1 - w_1)\partial q_1} = \frac{\partial^2 \pi_2}{\partial(p_1 - w_1)\partial q_2} = 0$$

Therefore, nor q_1^{*VMI}, neither q_2^{*VMI}, change as $(p_1 - w_1)$ changes (again, same applies to $(p_2 - w_2)$).

As a consequence,

$\exists\; s_2', (p_1 - w_1)'$ such that, for $s_2'' < s_2'$, $(p_1 - w_1)'' > (p_1 - w_1)'$ implies $q_1^{*RMI} \geq q_1^{*VMI}$ and $q_2^{*RMI} \leq q_2^{*VMI}$, with analogous results for $(p_2 - w_2)$.

(iv) (b)

We know, from the proof of Proposition (iv) (a), that, as $s_j \to 0$, and $(p_i - w_i) \to \infty$, $q_i^{*RMI} \to \infty$ and $q_j^{*RMI} \to 0$.

Also,

$$\frac{\partial \pi_j^{RMI_iVMI_j}}{\partial q_j} = G_{i_{qj}}^{RMI_jVMI_i} = (w_j - c_j)(\frac{\partial S_j}{\partial q_j}) - (h_j)(\frac{\partial I_j}{\partial q_j}) - (s_j)(\frac{\partial Sh_j}{\partial q_j})$$

So a change in $(p_i - w_i)$ has no direct effect on $q_j^{*RMI_iVMI_j}$. However,

$$\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial q_i} = G_{R_{qi}}^{RMI_iVMI_j} = [(p_i - w_i) - (p_j - w_j)](\frac{\partial S_i}{\partial q_i}) - h_i(\frac{\partial I_i}{\partial q_i}) - s_i(\frac{\partial Sh_i}{\partial q_i})$$

So $(p_i - w_i) \to \infty$ does make $q_i^{*RMI_iVMI_j} \to \infty$. This, in turn has an effect on $q_j^{*RMI_iVMI_j}$ because, as we know from Proposition 3, an increase in $q_i^{*RMI_iVMI_j}$ decreases $q_j^{*RMI_iVMI_j}$ and vice-versa, because the cross-partial derivatives are negative.

Therefore, $(p_i - w_i) \to \infty$ does make $q_i^{*RMI_iVMI_j} \to \infty$, and $q_j^{*RMI_iVMI_j}$ decrease. However,

as long as $(w_j - c_j) > 0$, $q_j^{*RMI_iVMI_j} > 0$. Finally, as long as $q_j^{*RMI_iVMI_j} > q_j^{*RMI}$, $q_i^{*RMI_iVMI_j} < q_i^{*RMI}$ because, under RMI, product *i* sees more indirect demand.

As a consequence,

$\exists \ s_j', (p_i - w_i)'$ such that, for $s_j'' < s_j'$, $(p_i - w_i)'' > (p_i - w_i)'$ implies $q_j^{*RMI_iVMI_j} > q_j^{*RMI}$ and $q_i^{*RMI_iVMI_j} < q_i^{*RMI}$.

(v)

Under RMI,

$$\frac{\partial^2 \pi_R^{RMI}}{\partial(p_j - w_j)\partial q_j} = \frac{\partial\, G_{R_{qj}}^{RMI}}{\partial p_j} = \frac{\partial\, S_2}{\partial q_j} \geq 0$$

So, as $(p_j - w_j) \rightarrow \infty \;\; q_j^{*RMI} \rightarrow \infty$.

Under the mixed scenario, however,

$$\frac{\partial\, \pi_R^{RMI_iVMI_j}}{\partial q_i} = [(p_i - w_i) - (p_j - w_j)](\frac{\partial\, S_i}{\partial q_i}) - h_i(\frac{\partial\, I_i}{\partial q_i}) - s_i(\frac{\partial\, Sh_i}{\partial q_i})$$

but

$$\frac{\partial\, \pi_j^{RMI_iVMI_j}}{\partial q_j} = G_{i_{qj}}^{RMI_jVMI_i} = (w_j - c_j)(\frac{\partial\, S_j}{\partial q_j}) - (h_j)(\frac{\partial\, I_j}{\partial q_j}) - (s_j)(\frac{\partial\, Sh_j}{\partial q_j})$$

So, as $(p_j - w_j) \rightarrow \infty$, $q_i^{*RMI_iVMI_j}$ decreases, diverting more demand intro product *j*, and making $q_j^{*RMI_iVMI_j}$ go up. However, even if $q_i^{*RMI_iVMI_j} = 0$, $q_j^{*RMI_iVMI_j}$ would be finite.

Thus,

$\exists\; (p_j - w_j)'$ such that, for $(p_j - w_j)'' > (p_j - w_j)'$ implies $q_j^{*RMI} \geq q_j^{*RMI_iVMI_j}$.

Proof of Proposition *5*

(*i*)(a)

Proposition 4 (iii) states that if both $(w_i - c_i)$ and $(w_j - c_j)$ are large enough, $q^{*RMI} \leq q^{*VMI}$ for both products, and $q_j^{*RMI} \leq q_j^{*RMI_iVMI_j}$. It can also be shown that, in this case, $q_i^{*RMI} \geq q_i^{*RMI_iVMI_j}$, because under RMI_iVMI_j the fact that manufacturer *j* will stock more makes *i* see less indirect demand. However, although as $(w_i - c_i) \rightarrow \infty$, $q_i^{*VMI} \rightarrow \infty$, $q_i^{*RMI_iVMI_j}$ does not change with $(w_i - c_i)$, and no matter how large $(w_j - c_j)$, even if $q_j^{*RMI_iVMI_j} \rightarrow \infty$, $q_i^{*RMI_iVMI_j}$ does not go to infinity (it merely converges to the solution to a simple newsvendor without substitution). But, if $(p_j - w_j) \approx (p_i - w_i)$, then diverting demand either way does not increase the retailer's sales, and therefore the fact that as $(w_i - c_i)$ and $(w_j - c_j)$ become larger, the manufacturers will stock more of both products under VMI for both, dominates because: (a) it decreases the retailer's shortage costs, and (b) under VMI, the retailer does not pay holding costs.

(i) (b)

If $\alpha_{ij} = \alpha_{ji} = \alpha = 0$, Proposition 4 ($i$) states that the quantities stocked will be the same under the three scenarios. Thus, the retailer is better off doing VMI for both products, effectively pushing holding costs upstream to the manufacturers.

(ii)

Similarly, Proposition 4 (ii) states that if $(w_i - c_i)$, s_i, $(w_j - c_j)$, and s_j are small enough, then $q^{*RMI} \geq q^{*VMI}$, with similar results for asymmetric cases, and $q_j^{*RMI} \geq q_j^{*RMI_iVMI_j}$. In fact, the quantities stocked of the product being under VMI tend to zero as the corresponding manufacturer's margins and shortage costs tend to zero.

Under VMI for both products, as both stocking quantities tend to zero, so do the retailer's profits. Under RMI_iVMI_j, $q_j^{*RMI_iVMI_j}$ tends to zero, which makes the retailer (weakly) worse off than under RMI for both products. (The retailer could be indifferent between RMI and RMI_iVMI_j if stocking very little of *j* was good for him anyway because *i* was lucrative enough to make diversion worth it. More about this will be explored in the proof of Proposition 5 (iii) (a).). If, in addition, $(p_j - w_j)$ and $(p_i - w_i)$ are large enough, then it will pay for the retailer assuming control of the products because the increase in sales would justify the extra inventory holding cost.

(iii) (a)

The proof has two parts:

Part I

(a.1) Under RMI, as the Proof of Propositon 4 (iv) shows, as $(p_i - w_i)$ tends to infinity, and s_j tends to zero, then $q_i^{*RMI} \to \infty$ and $q_j^{*RMI} \to 0$, because it pays for the retailer to understock product *j* and divert demand to product *i*.

(a.2) Under VMI for both products, the Proof of Proposition 4 (iii) shows that if $(w_i - c_i)$ and $(w_j - c_j)$ are large enough, $q^{*RMI} \leq q^{*VMI}$ for both products.

Now, because under VMI for both products product *j*'s stock does not tend to zero, in fact it tends to infinity as $(w_j - c_j)$ tends to infinity, the diversion of demand from product *j* to the more lucrative product *i* will be less and less under VMI for both products. If $(p_j - w_j)$ is small enough, $(p_i - w_i)$ is large enough, and h_i, h_j are small enough the extra sales of product *j*, and the holding cost savings under VMI for both will not offset the revenue lost because "diversion" from *j* to *i* is lower.

Then, $\pi_R^{*RMI} \geq \pi_R^{*VMI}$.

Part II

The following setting, however, will make the retailer better off as long as $(p_i - w_i) \leq (w_i - c_i)$: assign product *i* to manufacturer *i* –*i.e.*, do VMI with product *i*. However, keep decision rights for product *j* –*i.e.*, do RMI for product *j*. Next, keep $q_j^{RMI_iVMI_j}$ low. Then, $q_i^{RMI_iVMI_j} \geq q_i^{RMI}$ because manufacturer *i* has larger margins than the retailer, and $q_j^{RMI_iVMI_j} \leq q_j^{RMI_i}$ because it may pay to induce manufacturer *i* to see even more indirect demand given that the retailer does not pay for holding costs for *i* and manufacturer *i* has larger margins than the retailer. Therefore, under RMI_jVMI_i

the retailer sees more sales of *i* than under RMI for both, and saves the holding costs on product *i*, the product with the higher stock.

Thus, $\pi_R^{*RMI_jVMI_i} > \pi_R^{*RMI}$.

(iii) (b)

The proof has two parts.

Part I

Under VMI for both, the proof of Proposition 4 (ii) shows that, as $(w_i - c_i)$, and s_i tend to zero, so does q_i^{*VMI} . On the other hand, the proof of Proposition 4 (iii) shows that, as $(w_j - c_j)$ tend to infinity, so does q_j^{*VMI} . So, by moving from RMI for both to VMI for both, the retailer (a) saves holding costs on product *i,j*, (b) sees less lost sales of product *j*, (c) sees more lost sales of product *i*. If $(w_j - c_j)$ is large enough, then (b) can dominate (c).

Thus, $\pi_R^{*RMI} \leq \pi_R^{*VMI}$.

Part II

Moving from VMI for both to RMI_iVMI_j the retailer, (a) pays more holding cost on product *i*, (b) sees less lost sales of product *i* (because, if $(w_i - c_i)$ is small enough, the retailers margins will be larger, and he will stock more than manufacturer *i*. If h_i is small enough, then (b) will offset (a).

Thus, $\pi_R^{*RMI_iVMI_j} \geq \pi_R^{*VMI}$.

Proof of Proposition *6:*

$$\frac{\partial\ \pi_R^{RMI}}{\partial\alpha_{ij}} = (p_i - w_i)(\frac{\partial\ S_i}{\partial\alpha_{ij}}) + (p_j - w_j)(\frac{\partial\ S_j}{\partial\alpha_{ij}}) - h_i(\frac{\partial\ I_i}{\partial\alpha_{ij}}) - h_j(\frac{\partial\ I_j}{\partial\alpha_{ij}})$$

But,

$$\frac{\partial\ S_i}{\partial\alpha_{ij}} = 0 \quad \frac{\partial\ S_j}{\partial\alpha_{ij}} \geq 0 \quad \frac{\partial\ I_i}{\partial\alpha_{ij}} = 0 \quad \frac{\partial\ I_j}{\partial\alpha_{ij}} \leq 0$$

So,

$$\frac{\partial\ \pi_R^{RMI}}{\partial\alpha_{ij}} \geq 0 .$$

This is true $\forall\ q_i^{RMI}$ and q_j^{RMI} .

Proof of Proposition *7.*

(i)

Netessine and Rudi (2003), showed that the first order condition for VMI can be written:

$$\frac{\partial\pi_1^{VMI}}{\partial q_1} = (w_1 - c_1)Pr(d_1^S > q_1) - (c_1 + h_1)Pr(d_1^S \leq q_1)$$

$$\frac{\partial\pi_2^{VMI}}{\partial q_2} = (w_2 - c_2)Pr(d_2^S > q_2) - (c_2 + h_2)Pr(d_2^S \leq q_2)$$

where, $d_1^s = d_1 + \alpha_{21}(d_2 - q_2)^+$ and $d_2^s = d_2 + \alpha_{12}(d_1 - q_1)^+$

$$\frac{\partial^2\pi_1^{VMI}}{\partial\alpha_{21}\partial q_1} = (w_1 - c_1)\frac{\partial Pr}{\partial\alpha_{21}}(d_1^S > q_1) - (c_1 + h_1)\frac{\partial Pr}{\partial\alpha_{21}}(d_1^S \leq q_1) \geq 0$$

because $\frac{\partial Pr}{\partial \alpha_{21}}(d_1^S > q_1) \geq 0$ and $\frac{\partial Pr}{\partial \alpha_{21}}(d_1^S \leq q_1) \leq 0$.

And

$$\frac{\partial^2 \pi_2^{VMI}}{\partial \alpha_{21} \partial q_2} = 0$$

So, the direct effect of an increase in α_{21} is an increase in q_1^{*VMI}, and there is no direct effect on q_2^{*VMI}. However, this does not mean that q_2^{*VMI} will not change, since

$$\frac{\partial^2 \pi_2^{VMI}}{\partial q_1 \partial q_2} = (w_2 - c_2)\frac{\partial Pr}{\partial q_1}(d_2^S > q_2) - (c_2 + h_2)\frac{\partial Pr}{\partial q_1}(d_2^S \leq q_2) \leq 0$$

because $\frac{\partial Pr}{\partial q_1}(d_2^S > q_2) \leq 0$ and $\frac{\partial Pr}{\partial q_1}(d_2^S \leq q_2) \geq 0$.

Therefore, the direct effect of an increase in q_1^{*VMI} is a decrease in q_2^{*VMI}.

Finally,

$$\frac{\partial^2 \pi_1^{VMI}}{\partial q_2 \partial q_1} = (w_1 - c_1)\frac{\partial Pr}{\partial q_2}(d_1^S > q_1) - (c_1 + h_1)\frac{\partial Pr}{\partial q_2}(d_1^S \leq q_1) \leq 0$$

because $\frac{\partial Pr}{\partial q_2}(d_1^S > q_1) \leq 0$ and $\frac{\partial Pr}{\partial q_2}(d_1^S \leq q_1) \geq 0$.

So, a decrease in q_2^{*VMI} increases q_1^{*VMI}.

(ii)

We know that:

$$\frac{\partial \pi_R^{VMI}}{\partial q_i} = -(p_j - w_j)\alpha_{ij}\, Pr(d_i > q_i) + (p_i - w_i)\, Pr(d_i^S > q_i) + (p_j - w_j)\alpha_{ij}\, Pr(d_j^S > q_j, d_i > q_i)$$

So, if

$$(p_j - w_j)\alpha_{ij}\, Pr(d_i > q_i) \leq (p_i - w_i)\, Pr(d_i^S > q_i) + (p_j - w_j)\alpha_{ij}\, Pr(d_j^S > q_j, d_i > q_i)$$

then, $\dfrac{\partial\ \pi_R^{VMI}}{\partial q_i} \geq 0$,

and, therefore, as q_i^{*VMI} goes up so does π_R^{RMI}.

On the other hand, if

$$(p_i - w_i)\alpha_{ji} Pr(d_j > q_j) \geq (p_j - w_j) Pr(d_j^S > q_j) + (p_i - w_i)\alpha_{ji} Pr(d_i^S > q_i, d_j > q_j)$$

then, $\dfrac{\partial\ \pi_R^{VMI}}{\partial q_j} \leq 0$.

and, therefore, as q_j^{*VMI} goes down π_R^{RMI} goes up.

Since an increase in α_{ji} makes q_i^{*VMI} go up, and q_j^{*VMI} go down, this completes the proof.

Proof of Proposition *8*

(*i*)

Following the logic in Netessine and Rudi (2003), we can write the first order conditions for RMI_iVMI_j as:

$$\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial q_i} = -(p_j - w_j)\alpha_{ij} Pr(d_i > q_i) + (p_i - w_i) Pr(d_i^S > q_i) - (w_i + h_i) Pr(d_i^S < q_i)$$
$$+ (p_j - w_j)\alpha_{ij} Pr(d_j^S > q_j, d_i > q_i)$$

for the retailer, and

$$\frac{\partial \pi_j^{RMI_iVMI_j}}{\partial q_j} = (w_j - c_j) Pr(d_j^S > q_j) - (c_j + h_j) Pr(d_j^S \leq q_j)$$

for manufacturer *j*, where, $d_i^s = d_i + \alpha_{ji}(d_j - q_j)^+$ and $d_j^s = d_j + \alpha_{ij}(d_i - q_i)^+$

So,

$$\frac{\partial^2 \pi_R^{RMI_iVMI_j}}{\partial \alpha_{ij} \partial q_i} = -(p_j - w_j) Pr(d_i > q_i) + (p_j - w_j) Pr(d_j^S > q_j, d_i > q_i)$$

$$+(p_j - w_j)\alpha_{ij}\frac{\partial Pr(d_j^S > q_j, d_i > q_i)}{\partial \alpha_{ij}} \le 0 \text{, and}$$

$$\frac{\partial^2 \pi_j^{RMI_iVMI_j}}{\partial \alpha_{ij}\partial q_j} = (w_j - c_j)\frac{\partial Pr(d_j^S > q_j)}{\partial \alpha_{ij}} - (c_j + h_j)\frac{\partial Pr(d_j^S \le q_j)}{\partial \alpha_{ij}} \ge 0$$

Finally,

$$\frac{\partial^2 \pi_R^{RMI_iVMI_j}}{\partial q_j \partial q_i} = (p_i - w_i)\frac{\partial Pr(d_i^S > q_i)}{\partial q_j} - (w_i + h_i)\frac{\partial Pr(d_i^S < q_i)}{\partial q_j}$$
$$+(p_j - w_j)\alpha_{ij}\frac{\partial Pr(d_j^S > q_j, d_i > q_i)}{\partial q_j} \le 0$$

$$\frac{\partial^2 \pi_j^{RMI_iVMI_j}}{\partial q_i \partial q_j} = (w_j - c_j)\frac{\partial Pr}{\partial q_i}(d_j^S > q_j) - (c_j + h_j)\frac{\partial Pr}{\partial q_i}(d_j^S \le q_j) \le 0$$

(ii)

$$\frac{\partial^2 \pi_R^{RMI_iVMI_j}}{\partial \alpha_{ji}\partial q_i} = (p_i - w_i)\frac{\partial Pr(d_i^S > q_i)}{\partial \alpha_{ji}} - (w_i + h_i)\frac{\partial Pr(d_i^S < q_i)}{\partial \alpha_{ji}} \ge 0$$

$$\frac{\partial^2 \pi_j^{RMI_iVMI_j}}{\partial \alpha_{ji}\partial q_j} = (w_j - c_j)\frac{\partial Pr(d_j^S > q_j)}{\partial \alpha_{ji}} - (c_j + h_j)\frac{\partial Pr(d_j^S \le q_j)}{\partial \alpha_{ji}} = 0$$

and the cross-partial derivatives are just as in the proof of Proposition 8 (*i*).

(iii) We know, from Proposition 8 (*i*) and (ii) that,

(*i*) an increase in α_{ij} ,will make $q_i^{*RMI_iVMI_j}$ (weakly) decrease but $q_j^{*RMI_iVMI_j}$ increase (weakly),

(ii) an increase in α_{ji} , will make $q_i^{*RMI_iVMI_j}$ (weakly) increase but $q_j^{*RMI_iVMI_j}$ decrease (weakly),

How does this affect retailer's profits?

$$\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial q_i} = -(p_j - w_j)\alpha_{ij} Pr(d_i > q_i) + (p_i - w_i) Pr(d_i^S > q_i) - (w_i + h_i) Pr(d_i^S < q_i)$$
$$+ (p_j - w_j)\alpha_{ij} Pr(d_j^S > q_j, d_i > q_i)$$

$$\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial q_j} = -(p_i - w_i)\alpha_{ji} Pr(d_j > q_j) + (p_j - w_j) Pr(d_j^S > q_j)$$
$$+ (p_i - w_i)\alpha_{ji} Pr(d_i^S > q_i, d_j > q_j)$$

(a) if

$$(p_j - w_j)\alpha_{ij} Pr(d_i > q_i) - (p_j - w_j)\alpha_{ij} Pr(d_j^S > q_j, d_i > q_i) + (w_i + h_i) Pr(d_i^S < q_i)$$
$$\geq (p_i - w_i) Pr(d_i^S > q_i)$$

and

$$(p_i - w_i)\alpha_{ji} Pr(d_j > q_j) - (p_i - w_i)\alpha_{ji} Pr(d_i^S > q_i, d_j > q_j) \leq (p_j - w_j) Pr(d_j^S > q_j)$$

then $\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial \alpha_{ij}} \geq 0$, and $\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial \alpha_{ji}} \leq 0$

(b) if

$$(p_j - w_j)\alpha_{ij} Pr(d_i > q_i) - (p_j - w_j)\alpha_{ij} Pr(d_j^S > q_j, d_i > q_i) + (w_i + h_i) Pr(d_i^S < q_i)$$
$$\leq (p_i - w_i) Pr(d_i^S > q_i)$$

and

$$(p_i - w_i)\alpha_{ji} Pr(d_j > q_j) - (p_i - w_i)\alpha_{ji} Pr(d_i^S > q_i, d_j > q_j) \geq (p_j - w_j) Pr(d_j^S > q_j)$$

then $\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial \alpha_{ij}} \leq 0$, and $\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial \alpha_{ji}} \geq 0$.

Proof of Proposition *9*

(*i*)

If $\alpha = 0$,

Under RMI, the Retailer solves

$$\frac{\partial \pi_R^{RMI}}{\partial q_i} = (p_i - w_i) Pr(d_i > q_i) - (w_i + h_i) Pr(d_i \le q_i)$$

Under VMI, the Manufacturers solve

$$\frac{\partial \pi_i^{VMI}}{\partial q_i} = (w_i - c_i) Pr(d_i > q_i) - (c_i + h_i) Pr(d_i \le q_i)$$

Although both first order conditions look like a newsvendor problem, the economic parameters are different: under RMI, the newsvendor's margin is $(p_i - w_i)$, and the overstocking cost is $(w_i + h_i)$, while under VMI the newsvendor's margin is $(w_i - c_i)$, and the understocking cost is $(c_i + h_i)$.

(ii)

See Proposition 5 (iii) in Netessine and Rudi (2003).

(iii) (a) and (iv)

Following Netessine and Rudi (2003), we can write the first order conditions for RMI as

$$\frac{\partial \pi_R^{RMI}}{\partial q_1} = -(p_2 - w_2)\alpha_{12} Pr(d_1 > q_1) + (p_1 - w_1) Pr(d_1^S > q_1) - (w_1 + h_1) Pr(d_1^S < q_1)$$
$$+ (p_2 - w_2)\alpha_{12} Pr(d_2^S > q_2, d_1 > q_1) - (w_2 + h_2)\alpha_{12} Pr(d_2^S < q_2, d_1 > q_1)$$

where, $d_1^s = d_1 + \alpha_{21}(d_2 - q_2)^+$ and $d_2^s = d_2 + \alpha_{12}(d_1 - q_1)^+$

Because of symmetry, we will omit all the sub-scripts.

$$\frac{\partial^2 \pi_R^{RMI}}{\partial (p-w) \partial q} = -\alpha Pr(d > q) + Pr(d^S > q) + \alpha Pr(d^S > q, d > q) \ge 0$$

since $Pr(d > q) \le Pr(d^S > q)$

So, as $(p - w) \to \infty, q^{*RMI} \to \infty$, but a change in $(w - c)$ does not affect q^{*RMI}.

As mentioned before, the VMI first order conditions are:

$$\frac{\partial \pi_1^{VMI}}{\partial q_1} = (w_1 - c_1) Pr(d_1^S > q_1) - (c_1 + h_1) Pr(d_1^S \leq q_1)$$

Once again, omitting all subscripts and derivating yields,

$$\frac{\partial^2 \pi_1^{VMI}}{\partial (w-c) \partial q} = Pr(d_1^S > q_1) \geq 0$$

So, as $(w-c) \to 0, q^{*VMI} \to 0$.

Therefore,

(1) $\exists \ (p-w)'$ such that $(p-w)'' > (p-w)'$ implies $q^{*RMI} \geq q^{*VMI}$.

(2) $\exists \ (w-c)'$ such that $(w-c)'' < (w-c)'$ implies $q^{*RMI} \geq q^{*VMI}$.

For asymmetric cases, the results would be:

(1) $\exists \ (p_i - w_i)'$ such that, for $(p_i - w_i)'' > (p_i - w_i)'$ implies $q_i^{*RMI} \geq q_i^{*VMI} \ i \in \{1,2\}$.

(2) $\exists \ (w_i - c_i)'$ such that, for $(w_i - c_i)'' < (w_i - c_i)'$ implies $q_i^{*RMI} \geq q_i^{*VMI} \ i \in \{1,2\}$.

(iii) (b)

The RMI part is just as the proof of Proposition 9 (a).

$$\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial q_i} = -(p_j - w_j)\alpha_{ij} Pr(d_i > q_i) + (p_i - w_i) Pr(d_i^S > q_i) - (w_i + h_i) Pr(d_i^S < q_i)$$
$$+ (p_j - w_j)\alpha_{ij} Pr(d_j^S > q_j, d_i > q_i)$$

$$\frac{\partial \pi_j^{RMI_iVMI_j}}{\partial q_j} = (w_j - c_j) Pr(d_j^S > q_j) - (c_j + h_j) Pr(d_j^S \leq q_j)$$

So, as $(w-c) \to 0, q_j^{*RMI_iVMI_j} \to 0$.

Therefore,

$\exists \ (w_i - c_i)'$ such that, for $(w_i - c_i)'' < (w_i - c_i)'$ implies $q_i^{*RMI} \geq q_i^{*RMI_iVMI_j} \ i \in \{1,2\}$.

(v)

From the derivative shown in the proof of Proposition 9 (iii) (b), it can be seen that, if $(p_j - w_j) \to \infty$, $q_i^{*RMI_iVMI_j}$ decreases (weakly) to attempt to deviate more demand to product *j*, while $q_j^{*RMI_iVMI_j}$ increases because *j* sees more demand. However, even if $q_i^{*RMI_iVMI_j} = 0$, $q_j^{*RMI_iVMI_j}$ is finite.

Therefore,

$\exists\ (p_j - w_j)'$ such that, for $(p_j - w_j)'' > (p_j - w_j)'$ implies $q_j^{*RMI} \geq q_j^{*RMI_iVMI_j}$ $i \in \{1,2\}$.

(vi) (a)

Again, the first order conditions for RMI are

$$\frac{\partial \pi_R^{RMI}}{\partial q_1} = -(p_2 - w_2)\alpha_{12} Pr(d_1 > q_1) + (p_1 - w_1) Pr(d_1^S > q_1) - (w_1 + h_1) Pr(d_1^S < q_1)$$
$$+ (p_2 - w_2)\alpha_{12} Pr(d_2^S > q_2, d_1 > q_1) - (w_2 + h_2)\alpha_{12} Pr(d_2^S < q_2, d_1 > q_1)$$

$$\frac{\partial \pi_R^{RMI}}{\partial q_2} = -(p_1 - w_1)\alpha_{21} Pr(d_2 > q_2) + (p_2 - w_2) Pr(d_2^S > q_2) - (w_2 + h_2) Pr(d_2^S < q_2)$$
$$+ (p_1 - w_1)\alpha_{21} Pr(d_1^S > q_1, d_2 > q_2) - (w_1 + h_1)\alpha_{21} Pr(d_1^S < q_1, d_2 > q_2)$$

Thus,

$$\frac{\partial^2 \pi_R^{RMI}}{\partial (p_1 - w_1)\partial q_1} = Pr(d_1^S > q_1) \geq 0$$

and

$$\frac{\partial^2 \pi_R^{RMI}}{\partial (p_1 - w_1)\partial q_2} = -\alpha_{21} Pr(d_2 > q_2) + \alpha_{21} Pr(d_1^S > q_1, d_2 > q_2) \leq 0$$

since

$$Pr(d_2 > q_2) \geq Pr(d_1^S > q_1, d_2 > q_2).$$

So, as $(p_1 - w_1) \rightarrow \infty, q_1^{*RMI} \rightarrow \infty$ and $q_2^{*RMI} \rightarrow 0$, essentially because it pays to divert more and more demand from product 2 into product 1. This is without loss of generality, and similar results apply for $(p_2 - w_2)$.

The first order conditions for VMI, as stated earlier, can be written:

$$\frac{\partial \pi_1^{VMI}}{\partial q_1} = (w_1 - c_1) Pr(d_1^S > Q_1) - (c_1 + h_1) Pr(d_1^S \leq Q_1)$$
$$\frac{\partial \pi_2^{VMI}}{\partial q_2} = (w_2 - c_2) Pr(d_2^S > Q_2) - (c_2 + h_2) Pr(d_2^S \leq Q_2)$$

and are, therefore, independent of $(p_i - w_i), i \in \{1,2\}$.

Thus, neither q_1^{*VMI} or q_2^{*VMI} changes as $(p_1 - w_1)$ or $(p_2 - w_2)$ change.

As a consequence,

$\exists \ (p_1 - w_1)'$ such that $(p_1 - w_1)'' > (p_1 - w_1)'$ implies $q_1^{*RMI} \geq q_1^{*VMI}$ and $q_2^{*RMI} \leq q_2^{*VMI}$, with analogous results for $(p_2 - w_2)$.

(vi) (b)

The RMI observations are just as in the proof of Proposition 9 (vi) (a).

Form the mixed case,

$$\frac{\partial \pi_R^{RMI_iVMI_j}}{\partial q_i} = -(p_j - w_j)\alpha_{ij} Pr(d_i > q_i) + (p_i - w_i) Pr(d_i^S > q_i) - (w_i + h_i) Pr(d_i^S < q_i)$$
$$+ (p_j - w_j)\alpha_{ij} Pr(d_j^S > q_j, d_i > q_i)$$

$$\frac{\partial \pi_j^{RMI_iVMI_j}}{\partial q_j} = (w_j - c_j) Pr(d_j^S > q_j) - (c_j + h_j) Pr(d_j^S \leq q_j)$$

So, as $(p_1 - w_1) \rightarrow \infty, q_i^{*RMI_iVMI_j} \rightarrow \infty$, and $q_j^{*RMI_iVMI_j}$ decreases (weakly) because of the negative cross partial derivatives, but does not tend to zero no matter how large $q_i^{*RMI_iVMI_j}$.

Thus,

$\exists\ (p_i - w_i)'$ such that $(p_i - w_i)'' > (p_i - w_i)'$ implies $q_j^{*RMI} \leq q_j^{*RMI_iVMI_j}$.

Once $q_j^{*RMI} \leq q_j^{*RMI_iVMI_j}$, then $q_i^{*RMI} \geq q_i^{*RMI_iVMI_j}$ because product *i* sees more indirect demand if both products are under RMI.

(vii) (a)

From the proof of Proposition 9 (iii) (a), we know that,

As $(p-w) \to 0$, $q^{*RMI} \to 0$, but a change in $(w-c)$ does not affect q^{*RMI}, and

as $(w-c) \to \infty$, $q^{*VMI} \to \infty$, but a change in $(p-w)$ does not affect q^{*VMI}.

Therefore,

(1) $\exists\ (p-w)'$ such that $(p-w)'' < (p-w)'$ implies $q^{*RMI} \leq q^{*VMI}$.

(2) $\exists\ (w-c)'$ such that $(w-c)'' > (w-c)'$ implies $q^{*RMI} \leq q^{*VMI}$.

For asymmetric cases, the results would be:

(1) $\exists\ (p_i - w_i)'$ such that, for $(p_i - w_i)'' < (p_i - w_i)'$ implies $q_i^{*RMI} \leq q_i^{*VMI}$ $i \in \{1,2\}$.

(2) $\exists\ (w_i - c_i)'$ such that, for $(w_i - c_i)'' > (w_i - c_i)'$ implies $q_i^{*RMI} \leq q_i^{*VMI}$ $i \in \{1,2\}$.

(vii) (b)

The RMI analysis is just as in the proof of Proposition 9 (vii) (a).

In the mixed case, it can be shown that:

As $(p-w)$ goes to zero, $q_i^{*RMI_iVMI_j}$ goes to zero, but $q_j^{*RMI_iVMI_j}$ grows (because the crosspartial derivatives are negative).

Also, as $(w-c)$ grows, so does $q_j^{*RMI_iVMI_j}$.

Therefore,

(1) $\exists\ (p-w)'$ such that $(p-w)'' < (p-w)'$ implies $q_j^{*RMI} \leq q_j^{*RMI_iVMI_j}$.

(2) $\exists$ $(w-c)'$ such that $(w-c)'' > (w-c)'$ implies $q_j^{*RMI} \leq q_j^{*RMI_i VMI_j}$.

For asymmetric cases, the results would be:

(1) $\exists$ $(p_j - w_j)'$ such that, for $(p_j - w_j)'' < (p_j - w_j)'$ implies $q_j^{*RMI} \leq q_j^{*RMI_i VMI_j}$.

(2) $\exists$ $(w_j - c_j)'$ such that, for $(w_j - c_j)'' > (w_j - c_j)'$ implies $q_j^{*RMI} \leq q_j^{*RMI_i VMI_j}$.

Proof of Proposition *10*

The logic of Proposition 5 applies to the lost sales case, and therefore, a complete new proof will be omitted. We will, however, comment one change: in part (*i*) (b) , the condition of $\alpha_{ij} = \alpha_{ji} = \alpha = 0$ is slightly different: while, under backorders, the rule was, "if $\alpha_{ij} = \alpha_{ji} = \alpha = 0$, do VMI regardless of margin", under lost sales, the rule is "if either manufacturers' margin is small enough, and retailer's margin is large enough, and $\alpha_{ij} = \alpha_{ji} = \alpha = 0$, then do RMI for that product", and "if either manufacturer's margins is large enough (or retailer's margins are small enough), then do VMI for that product". In essence, absent substitution, under lost sales it will be best for the retailer to simply assign decision rights over quantities to the party having the higher margins, provided that these margins are high enough, and, in the case of RMI, that the corresponding increase in sales offsets taking care of holding costs. Note that this was not true when backorders where assumed: in the full backorders case, if $\alpha_{ij} = \alpha_{ji} = \alpha = 0$ all quantities are equal in the three scenarios, and thus VMI always dominates, regardless of margins.

Notes

[i] In this paper, following tradition, the upstream parties, *i.e.* the manufacturers, will be considered female, and the downstream parties, *i.e.* the retailer, will be considered male.
[ii] In our newspaper example, if newspapers buy back unsold inventory, it can be assumed that this is equivalent to the retailer facing a lower holding cost. Note that, although the newspaper may buy back at full wholesale price, holding costs (or equivalently, salvage value) for the newsvendor is still non zero. This is because of either opportunity cost of space or, as we learned in our interaction with a large metropolitan newspaper, because some newsvendors have to pay for the garbage that unsold newspapers generate.
[iii] In the most general case that allows for a goodwill loss on customers buying their second choice, the result would be "as long as the goodwill loss on customers switching from i to j is lower than the margin on product j, an increase in α_{ij} will make the retailer better off".