WHO SHOULD TAKE OVER OPERATIONS IN A SUPPLY CHAIN? THE EFFECT OF RETAILER, MANUFACTURER EFFORTS, AND SPECIFICITY.

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Abstract

When is having no previous contract better than having one, if ever? When should a manufacturer/retailer take over part (or all) of the other party's operation, instead of considering just a contract? How about the opposite situation? This paper responds these questions in the context of a Supply Chain under a single period scenario where both the manufacturer and the retailer can exert uncontractible effort that may or may not be specific to the relationship. We thus contribute to the Supply Chain Literature with a new application of the economics' incomplete contract approach to vertical integration.

Keywords

Newsvendor, contracts, incentives, Vendor Managed Inventory, retailer effort, manufacturer effort, vertical integration.

1. Introduction

In Supply Chains where both parties can exert non-contractible effort, when is having no previous contract better than having one, if ever? When should a manufacturer/retailer take over part (or all) of the other party's operation, instead of considering just a contract? How about the opposite situation? While we have been studying contract theory in the context of supply chains for some time now -arguably, since Pasternak's 1986 seminal paper about buybacks-, the focus has been on a rather different question, that could be summarized as: What is the best contract in a Supply Chain, under different circumstance? Willamson (1991) describes three types of interaction between firms: markets (*i.e.* no contracts), vertical integration, and hybrids (*i.e.* contracts, where most of the supply chain contract theory has focused). We feel that a full discussion about the proper place of contracts within Supply Chains among the three types of interaction is due. In this paper, we take a first step on this direction, by adapting some of the incomplete contract ideas from economics to a Supply Chains context.

We study this topic in the context of a Supply Chain under a single period scenario where both the manufacturer and the retailer can exert uncontractible effort that may or may not be specific to the relationship. We model effort as having three effects: (a) increasing demand for the product being traded between manufacturer and retailer, (b) increasing revenues of other activities not directly related to the product being traded –which could be understood as economies of scope-, and (c) increasing the opportunities that the parties would have if they did not trade. We find that, (1) when effort specificity is low, no previous contract is best –*i.e.* supply chain profits are (weakly) larger than if a contract is signed before effort is exerted, or than if either party takes over the other party's operations-; (2) when the manufacturer/retailer can take over the other party's operations without an efficiency loss, and can internalize the extra opportunities generated by the activities performed for the relationship, then they should do so –*i.e.* this alternative dominates other alternatives-; and, finally, we study a numerical example where (3) a contract followed by traditional Retailer Managed Inventory (RMI) is optimal –*i.e.* is better than no contract and than any party taking over the other party's activities- when effort specificity is high, and taking over the other party's operations either implies an efficiency loss or that economies of scope are not fully internalized.

The rest of the paper is organized as follows: Section 2 provides a Literature Review. Section 3 presents the models for each scenario. 3.1 Specifies the General Demand and Effort Assumptions, 3.2 Formulates the First Best Case, and 3.2.1 presents Comparative Statics. 3.3 Formulates the Case when Effort is Exerted without a Previous Contract, 3.3.1, shows the Comparative Statics for this case. 3.4 Introduces the case when a Contact is Signed Before Effort is Exerted, 3.4.1 models a case of Full Commitment and Retail Managed Inventory, 3.4.1.1, the Comparative Statics, for this case. 3.4.2. Models the case that allows for Renegotiation and Retail Managed Inventory, 3.4.2.1, the corresponding Comparative Statics. 3.4.3 models what happens when the Manufacturer Takes Over, 3.4.3.1, its Comparative Statics. 3.4.4 models the case when the Retailer Takes Over, while 3.4.4.1 evaluates the Comparative Statics of this case. Section 4 compares all cases, 4.1 presents a numerical example, and Section 5 concludes the paper.

2 Literature Review

While multiple scholars in economics, marketing and operations management have examined contracts between a single manufacturer and retailer in a supply chain, we are unaware of any other paper that deals with the kind of questions we posed in the introduction within a supply chain contracting framework. See Cachon (2002), Tsay and Agrawal (2002), Lariviere (1999), and Tsay, Nahmias, and Agrawal (1999) for literature reviews on supply chain contracts.

A number of papers model retailer efforts, -*e.g.*, Cachon and Lariviere (2002), Taylor (2000) and Netessine and Rudi (2001). There are considerably less papers that model manufacturer's efforts within the supply chain contracting literature. Kraiselburd, Narayanan and Raman (2004), compare RMI to VMI in the presence of competing, substitute products, when the manufacturer can exert observable but not verifiable effort. There are even fewer supply chain contract paper that assumes that uncontractible efforts are possible both at the retailer and manufacturer level. Corbett and DeCroix (2001), have a model that does consider this, but, in their paper, effort benefits one firm but hurts the other. A stream of papers discuss quality issues in supply chains. For example, in Baiman, Fischer and Rajan (2000), the supplier can exert effort that improves quality, and the buyer can exert testing effort. However, as mentioned in the introduction, most contracting papers in the supply chain literature are more concerned with finding a contract that achieves first best, than on describing the full range of choices that starts with an arms-length transaction and ends in vertical integration.

The broad question of what happens when a firm takes over control of a set of activities (vs just buying from the market) can be traced to Coase (1937). The main ideas for this paper where inspired by Grossman and Hart (1986), Hart and Moore (1990), and Hart (1989), but mostly by Hart (1995). The main difference with Hart *et al* is that, while their ideas point to a theory of Vertical Integration where the "inalienability of human capital" an property rights are key, (a) our model is not necessarily about Vertical Integration, because we allow for either the retailer of manufactures to take over part (but not all) of the other party's activities, (b) in our paper, the outside opportunities that the parties have can be the result of the firm, rather than its manager/owner "walking out" of the deal, (c) our model incorporates economies of scope or externalities to the deal in question while Hart *et al* do not specifically model these, (d) our model is concerned with Supply Chain issues while Hart *et al* model generic investments without any stocking decisions, (e) in our model, the case where no contract is better than a contract or integration is not directly related to the issue of renegotiation, while in Hart and Moore this is key, (f) finally, our paper considers *effort* specificity rather than *asset* specificity –although efforts can generate intangible assets this need not be the case-, and thus our ideas are more concerned with a firm taking over certain activities rather than ownership of assets. However, despite the differences, our model can be considered an application of Hart and Moore's ideas about contract incompleteness to a Supply Chains framework.

Che and Hausch (1999) model a situation that bears some similarities with ours. In their model, they allow for non contractible investments by the parties that benefit both the seller and the buyer, just as this paper does. The main differences between Che and Hausch (1999) and this paper are that (1) in the former quantities are contractible, while in the latter model they are not[i], (2) in their model, contract terms can be realized after randomness is realized, while our model requires quantities to be shipped and sold before the season starts, and (3) we model specifically inventory while their investments are generic. Because of these differences, in Che and Hausch (1999), (a) a contract without renegotiation can achieve first best, while in this paper we do not find such contract, and (b) depending on the seller and buyer's bargaining share, renegotiation can make the parties worse off *ex ante*, while in our model renegotiation is always good *ex ante*. Despite these differences, both papers find that no contract can sometimes be better than a contract.

Finally, the main ideas about specificity and the hold up problem originate from Williamson (1979, 1991), and Klein, Crawford, and Alchian (1978). Just as Williamson predicted, our model states that, absent specificity, no contract is best –*i.e.* the market mechanism should prevail-. However, we contribute to his ideas by providing more detail about the rest of the spectrum –*i.e.* from hybrid forms to vertical integration- in the context of supply chains –*i.e.* when inventory considerations matter-. Beyond specificity, we show how the decision to take over the other party's operations also depends on the efficiency of the party taking over (vs the existing party) and the extent to which extra revenue opportunities generated by the operation can be internalized by the party taking over.

3. The Model

3.1 General Demand and effort assumptions

Let there be two risk neutral firms in a supply chain: a manufacturer who sells a product to a retailer. Both parties are also involved in other activities not specified in this model. The product is sold by the retailer during the period of interest at an exogenously determined price r. The manufacturer produces the widget at a cost of c. There is no time value of money and no inventory holding costs, all units left at the end of the period are sold at a marked down price s. Each time end demand is unmet, the retailer incurs a goodwill loss of g dollars. When the retailer and manufacturer sign a contract, the shape of the contract is limited to a wholesale price w, and a lump-sum money transfer T (that could go either way), with $c \leq w \leq r$ This is, in essence, is a "single period" model (although we do allow for decisions to be made at two points in time). Demand for the product is denoted by $X = D + \mu_1 em + \alpha_1 er$, where D is a random variable with a strictly increasing and continuous cumulative distribution; em is the amount of effort (including time and resources) that the manufacturer expends to increase demand for the product; $\mu_1 > 0$ is the sensitivity of demand to such effort; er is the amount of effort (including time and resources) that the retailer expends to increase demand for the product; $\alpha_1 > 0$ is the sensitivity of demand to such effort. Note that we have separated demand in two components: a deterministic component that depends on *er* and *em*, and a random component that is independent of efforts. We also assume that these efforts are observable but not verifiable, *i.e.* uncontractible. Each effort has an opportunity cost equal to the square of itself[ii]- that captures the notion that the marginal cost of effort is increasing in the effort. Let Q_{tot} represent the stocking quantity; we use the symbol $Q = Q_{tot} - \mu_1 em - \alpha_1 er$ to denote the stocking quantity, less the demand generated by the manufacturer's and retailer's efforts. We refer to Q as the "random component" of the stocking quantity. The order for Q_{tot} must be placed before *D* is realized –*i.e.* the parties do not know demand at the time of trade-. In addition to generating demand for the product, efforts have another two effects: (1) they generate extra revenues that only capitalize if the trade between the manufacturer and the retailer takes place –this can be regarded as economies of scope generated by the sale/production of the widget-, and (2) they increase the revenues that each party would make if trade did not happen –*i.e.*, the outside opportunities should the party not accept the contract. If trade happens, the retailer's marginal increase in revenues from exerting effort effort is $\alpha_1(r-c)+\alpha_2$ where α_2 measures the effect in dollar terms of the extra opportunities mentioned in (1) , and the manufacturer's marginal increase in revenues from exerting effort is $\mu_1(r-c)+\mu_2$, where μ_2 measures the effect in dollar terms of the extra opportunities mentioned in (1). $\mu_3(w_0-c)$ and $\alpha_3(r-w_0)$ reflect, correspondingly, the maufacturer's and the retailer's marginal increase in revenue per unit of effort if trade does not happen and both parties are forced to sell/buy their products from a generic market at price w_0, with $c < w_0 < r$. Finally, let $\mu_1(r-c)+\mu_2 \geq \mu_3(w_0-c)$ and $\alpha_1(r-c)+\alpha_2 \geq \alpha_3(r-w_0)$ which ensures that it is efficient for the parties to trade.

3.2 First Best Case Formulation

In a first best world, the problem is:

$$\underset{em,er,Q}{Max} \quad \pi_{FB} = \int_0^Q (D(r-c)+(Q-D)(s-c))f[D]dD + \int_Q^{\lambda} (Q(r-c)+(D-Q)(g))f[D]dD$$
$$+\mu_1 em(r-c)+\mu_2 em+\alpha_1 er(r-c)+\alpha_2 er - em^2 - er^2$$

Which yields:

$$em^*_{FB} = \frac{1}{2}((r-c)\mu_1+\mu_2),$$

$$er^*_{FB} = \frac{1}{2}((r-c)\alpha_1+\alpha_2), \text{ and}$$

$$Q^*_{FB} = F^{-1}[\frac{(r-c-g)}{(r-s-g)}],$$

which is the solution to a standard newsvendor problem.

3.2.1 Comparative statics

(a) $\frac{\partial em^*_{FB}}{\partial \mu_1} = \frac{1}{2}(r-c)$ (b) $\frac{\partial em^*_{FB}}{\partial \mu_2} = \frac{1}{2}$ (c) $\frac{\partial em^*_{FB}}{\partial \mu_3} = 0$

(d) $\frac{\partial em^*_{FB}}{\partial \alpha_1} = 0$ (e) $\frac{\partial em^*_{FB}}{\partial \alpha_2} = 0$ (f) $\frac{\partial em^*_{FB}}{\partial \alpha_3} = 0$

(g) $\frac{\partial er^*_{FB}}{\partial \mu_1} = 0$ (h) $\frac{\partial er^*_{FB}}{\partial \mu_2} = 0$ (i) $\frac{\partial er^*_{FB}}{\partial \mu_3} = 0$

(j) $\frac{\partial er^*_{FB}}{\partial \alpha_1} = \frac{1}{2}(r-c)$ (k) $\frac{\partial er^*_{FB}}{\partial \alpha_2} = \frac{1}{2}$ (l) $\frac{\partial er^*_{FB}}{\partial \alpha_3} = 0$

(m) $\frac{\partial Q^*_{FB}}{\partial \alpha_1} = \frac{\partial Q^*_{FB}}{\partial \mu_1} = \frac{\partial Q^*_{FB}}{\partial \alpha_2} = \frac{\partial Q^*_{FB}}{\partial \mu_2} = \frac{\partial Q^*_{FB}}{\partial \alpha_3} = \frac{\partial Q^*_{FB}}{\partial \mu_3} = 0$.

In a first best world, the manufacturer's efforts depend on how revenues are impacted if trade happens -results (a) and (b)-, but both the retailer's sensitivity of demand to efforts -results (d), (e) and (f)-and what could have happened if trade did not occur –result (c)- are irrelevant to the manufacturer. Similarly, the retailer cares about how revenues are impacted if trade happens –results (j), and (k)-, but the manufacturer's sensitivity of demand to efforts –results (g), (h), and (i), or what could have happened if trade did not occur –result (l)-, are irrelevant. Finally, first best efforts are irrelevant to Q, *i.e.* to the part of Q_{total} that responds to the randomness in demand. As we will see, the results of this section will not be exactly replicated under most realistic scenarios: each modeled case will have strengths and weaknesses given by inherent tradeoffs that will become apparent when studying comparative statics.

3.3 The Case When Effort is Exerted with No Previous Contract

For this section, assume that, just as figure 1 shows, effort is first exerted at $t = 0$ by the retailer and manufacturer without any contract, then, at $t = 1$, a contract is signed that agrees on a transfer price w and lump sum money transfer T, and finally, if the parties agree, at $t = 2$ trade happens. Figure 2 depicts what party exerts efforts, and who orders Q_{tot}

Figure 1: No previous contract timing

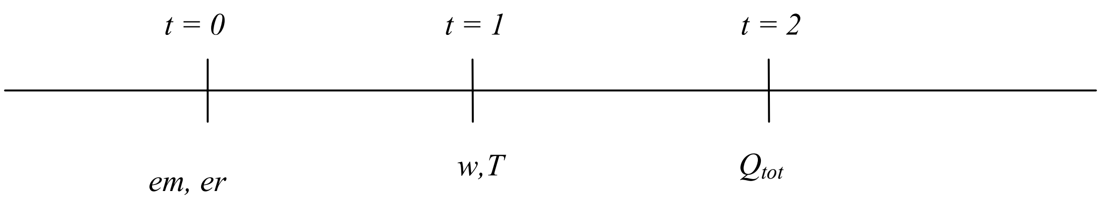
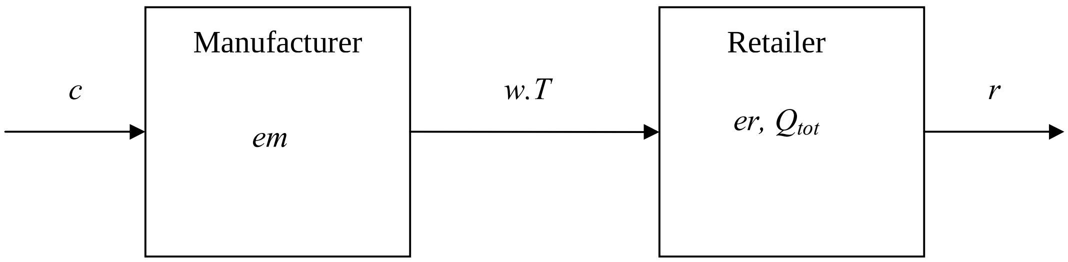


Figure 2: No previous contract structure



Note: as it will become clear later, who actually chooses Q is irrelevant in this scenario.

At $t = 1$, the cost of effort is sunk.

If no trade happens, the manufacturer's *ex post* profits are:

$$\pi_{NTMAN} = \mu_3 em(w_0 - c)$$

and the retailer's *ex post* profits are:

$$\pi_{NTRET} = \alpha_3 er(r - w_0)$$

if trade happens, total *ex post* supply chain profits are:

$$\pi_{NCTRADE} = \int_0^Q (D(r-c)+(Q-D)(s-c))f[D]dD + \int_Q^\lambda (Q(r-c)+(D-Q)(g))f[D]dD$$
$$+\mu_1 em(r-c)+\mu_2 em+\alpha_1 er(r-c)+\alpha_2 er$$

Note that, because effort has been sunk at $t = 1$, the retailer and manufacturer can agree to set $w = c$ and split their income via T[iii], thus avoiding double marginalization in Q.

Therefore, the gains from trade are:

$$\{\int_0^Q (D(r-c)+(Q-D)(s-c))f[D]dD + \int_Q^\lambda (Q(r-c)+(D-Q)(g))f[D]dD$$
$$+\mu_1 em(r-c)+\mu_2 em+\alpha_1 er(r-c)+\alpha_2 er\}-(\mu_3 em(w_0-c)+\alpha_3 er(r-w_0))$$

Now, assume that both parties split the gains from trade, and that the manufacturer gets $0 \le \theta \le 1$.

At $t = 0$, the manufacturer's *ex ante* problem is:

$$\underset{em}{Max} \quad \pi_{NCMAN} = \mu_3 em(w_0-c)-em^2 +$$
$$\theta(\int_0^Q (D(r-c)+(Q-D)(s-c))f[D]dD + \int_Q^\lambda (Q(r-c)+(D-Q)(g))f[D]dD$$
$$+\mu_1 em(r-c)+\mu_2 em+\alpha_1 er(r-c)+\alpha_2 er-\mu_3 em(w_0-c)+\alpha_3 er(r-w_0))$$

which yields:

$$em^*_{NC} = \frac{1}{2}(\theta((r-c)\mu_1+\mu_2)+(1-\theta)(w_0-c)\mu_3)$$

Correspondingly, the retailer gets $1-\theta$. Thus, at time $t = 0$, the retailer's *ex ante* profits are:

$$\underset{er}{Max} \quad \pi_{NCRET} = \alpha_3 er(r-w_0)-er^2 +$$
$$(1-\theta)(\int_0^Q (D(r-c)+(Q-D)(s-c))f[D]dD + \int_Q^\lambda (Q(r-c)+(D-Q)(g))f[D]dD$$
$$+\mu_1 em(r-c)+\mu_2 em+\alpha_1 er(r-c)+\alpha_2 er-\mu_3 em(w_0-c)+\alpha_3 er(r-w_0))$$

which yields

$$er_{NC}^{*} = \frac{1}{2}((1-\theta)((r-c)\alpha_1 + \alpha_2) + \theta(r-w_0)\alpha_3)$$

Finally, because $w = c$, it can be shown that, at $t = 2$,

$$Q_{FB}^{*} = Q_{NC}^{*}.$$

However,

$$Q_{totalFB}^{*} \geq Q_{totalNC}^{*}$$

because the efforts are different.

3.3.1 Comparative statics[iv]

(a) $\frac{\partial em_{NC}^{*}}{\partial \mu_1} = \frac{1}{2}\theta(r-c)$ (b) $\frac{\partial em_{NC}^{*}}{\partial \mu_2} = \frac{1}{2}\theta$ (c) $\frac{\partial em_{FB}^{*}}{\partial \mu_3} = (1-\theta)(w_0-c)$

(d) $\frac{\partial em_{NC}^{*}}{\partial \alpha_1} = 0$ (e) $\frac{\partial em_{NC}^{*}}{\partial \alpha_2} = 0$ (f) $\frac{\partial em_{NC}^{*}}{\partial \alpha_3} = 0$

(g) $\frac{\partial er_{NC}^{*}}{\partial \mu_1} = 0$ (h) $\frac{\partial er_{NC}^{*}}{\partial \mu_2} = 0$ (i) $\frac{\partial er_{NC}^{*}}{\partial \mu_3} = 0$

(j) $\frac{\partial er_{NC}^{*}}{\partial \alpha_1} = \frac{1}{2}(1-\theta)(r-c)$ (k) $\frac{\partial er_{NC}^{*}}{\partial \alpha_2} = \frac{1}{2}(1-\theta)$ (l) $\frac{\partial er_{NC}^{*}}{\partial \alpha_3} = \theta(r-w_0)$

(m) $\frac{\partial Q_{NC}^{*}}{\partial \alpha_1} = \frac{\partial Q_{NC}^{*}}{\partial \mu_1} = \frac{\partial Q_{NC}^{*}}{\partial \alpha_2} = \frac{\partial Q_{NC}^{*}}{\partial \mu_2} = \frac{\partial Q_{NC}^{*}}{\partial \alpha_3} = \frac{\partial Q_{NC}^{*}}{\partial \mu_3} = 0$

In this scenario, a number of things differ from the first best case. In fact, the manufacturer's effort now depends on:

(1) how revenues are impacted if trade happens, but the effect of effort is "watered down" by the splitting of revenues that happens after effort is exerted (via θ) -results (a) and (b)-, and

(2) what could have happened if trade did not occur, which was irrelevant to the manufacturer in a first best situation, –result (c)-.

Similarly, the retailer cares about:

(1) how revenues are impacted if trade happens "watered down", again, by (1-θ) –results (j), and (k)-, and

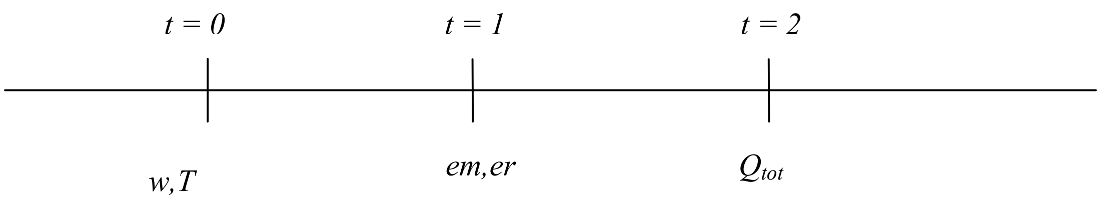
(2) what could have happened if trade did not occur, which was irrelevant to the retailer in a first best situation, –result (c)-.

Finally, just as in first best, efforts are irrelevant to Q, *i.e.* to the part of Q_{total} that responds to the randomness in demand.

The comparative statics show how, absent any contract, both manufacturer and retailer have to worry that once their effort has been sunk, negotiations may break, which creates a potential hold up problem. Thus, in this scenario, part of the effort is exerted to improve the parties negotiation power (which is directly related to the outside opportunities), and not directly to increase the revenues if trade happens.

3.4 The Cases when a Contract is Signed Before Effort is Exerted.

Figure 3: First contract, then effort timing.

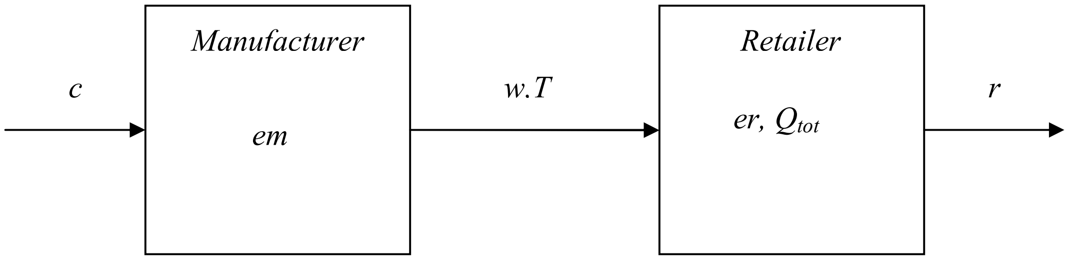


In this section, we will examine situations with the timing described in figure 3: here, the parties agree on a contract at $t = 0$ that specifies w (if applicable) and T, a lump-sum money transfer between the parties. This change may be able to mitigate the hold up problem, because the parties are guaranteed the transfer price and lump sum money transfer in advance, before any non contractible, relationship-specific efforts are exerted.

From now on, we will assume in our formulations that the manufacturer proposes the contract. As it will become clear later, if the retailer, instead of the manufacturer, proposes the contract, the only endogenous variable that will change is T, *i.e.* the efforts and quantities do not depend on what party is proposing the contract.

3.4.1 The Case of Full Commitment and Retailer Managed Inventory (FCRMI)

***Figure** 4: Retailer Managed Inventory with Contract Structure*



Here, after the contract is signed, each party exerts effort, and Q is ordered by the retailer. In this section, we will assume that what was signed at $t = 0$ will not be renegotiated later (thus, there is "full commitment"). Next section will discuss situations when renegotiation is possible, although in this paper we will not allow anyone to renege from a signed contract unless both parties agree (thus, in the next section we have "partial commitment").

To determine the contract at $t = 0$, the manufacturer's problem is:

$$\underset{w,T}{Max} \quad \pi_{MANFCRMI} = Q^*_{FCRMI}(w-c) + \mu_1 em^*_{FCRMI}(w-c) + \mu_2 em^*_{FCRMI} + \alpha_1 er^*_{FCRMI}(w-c) - (er^*_{FCRMI})^2 - T$$

Subject to:

(a) the retailer's participation constraint (PCR),

$$\int_0^{Q^*_{FCRMI}} (D(r-w) + (Q^*_{FCRMI} - D)(s-w)) f[D]\,dD + \int_{Q^*_{FCRMI}}^{\lambda} (Q^*_{FCRMI}(r-w) + (D - Q^*_{FCRMI})(g)) f[D]\,dD$$
$$+ \mu_1 em^*_{FCRMI}(r-w) + \alpha_1 er^*_{FCRMI}(r-w) + \alpha_2 er^*_{FCRMI} - (er^*_{FCRMI})^2 + T \geq \frac{1}{4}(r-w_0)^2 \alpha_3^2$$

(b) the manufacturer's incentive compatibility constraint (ICCM), and

$$em^*_{FCRMI} \in \underset{em}{Argmax}\, \pi_{FCMAN} = \quad \mu_1 em(w-c) + \mu_2 em + \alpha_1 er(w-c) - em^2$$

(c) the retailer's incentive compatibility constraint (ICCR):

$$Q^*_{FCRMI}, er^*_{FCRMI} \in \underset{er,Q}{Argmax}\, \pi_{FCRET}$$

where

$$\pi_{FCRET} = \int_0^Q (D(r-w)+(Q-D)(s-w))f[D]dD + \int_Q^\lambda (Q(r-w)+(D-Q)(g))f[D]dD$$
$$+\mu_1 em(r-w)+\alpha_1 er(r-w)+\alpha_2 er - er^2$$

Note: The PCR can be explained by noticing that, if the retailer walks out of the deal, her problem becomes $\underset{er}{Argmax} \quad \pi_{NTRET} = \alpha_3 er(r-w_0) - er^2$, which yields $er^*_{NT} = \frac{1}{2}\alpha_3(r-w_0)$, making the retailer profits $\pi^*_{NTRET} = \frac{1}{4}\alpha_3^2(r-w_0)^2$.

Since PCR will bind (if it did not, the manufacturer would simply decrease *T*),

$$T = \frac{1}{4}(r-w_0)^2\alpha_3^2 - (\int_0^{Q^*_{FCRMI}} (D(r-w)+(Q^*_{FCRMI}-D)(s-w))f[D]dD +$$
$$\int_{Q^*_{FCRMI}}^{\lambda} (Q^*_{FCRMI}(r-w)+(D-Q^*_{FCRMI})(g))f[D]dD +$$
$$\mu_1 em^*_{FCRMI}(r-w)+\alpha_1 er^*_{FCRMI}(r-w)+\alpha_2 er^*_{FCRMI} - (er^*_{FCRMI})^2)$$

So, the manufacturer's problem becomes:

$$\underset{w}{Max} \quad \pi_{MANFCRMI} = Q^*_{FCRMI}(w-c)+\mu_1 em^*_{FCRMI}(w-c)+\mu_2 em^*_{FCRMI}+\alpha_1 er^*_{FCRMI}(w-c)-(er^*_{FCRMI})^2$$
$$-\frac{1}{4}(r-w_0)^2\alpha_3^2 + (\int_0^{Q^*_{FCRMI}} (D(r-w)+(Q^*_{FCRMI}-D)(s-w))f[D]dD +$$
$$\int_{Q^*_{FCRMI}}^{\lambda} (Q^*_{FCRMI}(r-w)+(D-Q)(g))f[D]dD +$$
$$\mu_1 em^*_{FCRMI}(r-w)+\alpha_1 er^*_{FCRMI}(r-w)+\alpha_2 er^*_{FCRMI} - (er^*_{FCRMI})^2)$$

Subject to

$$em^*_{FCRMI} \in \underset{em}{Argmax}\, \pi_{FCRMIMAN} = \quad \mu_1 em(w-c) + \mu_2 em + \alpha_1 er(w-c) - em^2 \quad \text{(ICCM)}$$

and

$$Q^*_{FCRMI}, er^*_{FCRMI} \in \underset{er,Q}{Argmax}\, \pi_{FCRMIRET} \qquad \text{(ICCR)}$$

where, again,

$$\pi_{FCRMIRET} = \int_0^Q (D(r-w) + (Q-D)(s-w))f[D]dD + \int_Q^\lambda (Q(r-w) + (D-Q)(g))f[D]dD$$
$$+ \mu_1 em(r-w) + \alpha_1 er(r-w) + \alpha_2 er - er^2$$

Thus, w is set by the manufacturer to maximize total supply chain profits because she will be able to appropriate all profits in excess of the retailer's participation constraint. Note that, as we mentioned at the beginning of this section, a similar argument holds if the retailer, instead of the manufacturer, was proposing the contract. As our timing diagram indicate, once w and T are set, efforts happen. The solutions to the ICCs are:

$em^*_{FCRMI} = \frac{1}{2}((r-w)\mu_1 + \mu_2)$,

$er^*_{FCRMI} = \frac{1}{2}((r-w)\alpha_1 + \alpha_2)$, and

$Q^*_{FCRMI} = F^{-1}[\frac{(r-w-g)}{(r-s-g)}]$,

We are unable to find closed form solutions to the general problem described in this setting. However, we are still able either calculate or sign some comparative statics that can provide insights into this scenario.

3.4.1.1 Comparative Statics[v]

(a) $\frac{\partial em^*_{FCRMI}}{\partial \mu_1} > 0$ (b) $\frac{\partial em^*_{FCRMI}}{\partial \mu_2} = \frac{1}{2}$ (c) $\frac{\partial em^*_{FCRMI}}{\partial \mu_3} = 0$

(d) $\frac{\partial em^*_{FCRMI}}{\partial \alpha_1} < 0$ (e) $\frac{\partial em^*_{FCRMI}}{\partial \alpha_2} = 0$ (f) $\frac{\partial em^*_{FCRMI}}{\partial \alpha_3} = 0$

(g) $\dfrac{\partial er^*_{FCRMI}}{\partial \mu_1} < 0$ (h) $\dfrac{\partial er^*_{FCRMI}}{\partial \mu_2} = 0$ (i) $\dfrac{\partial er^*_{FCRMI}}{\partial \mu_3} = 0$

(j) $\dfrac{\partial er^*_{FCRMI}}{\partial \alpha_1} > 0$ (k) $\dfrac{\partial er^*_{FCRMI}}{\partial \alpha_2} = \dfrac{1}{2}$ (l) $\dfrac{\partial er^*_{FCRMI}}{\partial \alpha_3} = 0$

(m) $\dfrac{\partial Q^*_{FCRMI}}{\partial \mu_1} < 0$ (n) $\dfrac{\partial Q^*_{FCRMI}}{\partial \alpha_1} > 0$

(o) $\dfrac{\partial Q^*_{FCRMI}}{\partial \alpha_2} = \dfrac{\partial Q^*_{FCRMI}}{\partial \mu_2} = \dfrac{\partial Q^*_{FCRMI}}{\partial \alpha_3} = \dfrac{\partial Q^*_{FCRMI}}{\partial \mu_3} = 0$

The results in this section are driven by the fact that $\dfrac{\partial w^*_{FCRMI}}{\partial \alpha_1} < 0$, and $\dfrac{\partial w^*_{FCRMI}}{\partial \mu_1} > 0$, which is intuitive: if α_1 increases, it is optimal to increase the retailer's margin so he can exert more effort (thus exploiting the higher α_1); if, on the other hand, μ_1 increases, it is optimal to increase the manufacturer's margin to exploit this. However, because the manufacturer's margin is $(w-c)$ and the retailer's margin is $(r-w)$, increasing one party's margin implies reducing the other's. Thus, result (a) is a consequence of the manufacturer's higher margin on an increase in μ_1, and, correspondingly, (g) is a result of the retailer's lower margins on an increase of μ_1. Similarly, result (j) is a consequence of the retailer's higher margins on an increase in α_1, and (d) is a consequence of the manufacturer's lower margins on an increase in α_1. Finally, Q^*_{FCRMI} moves with the retailer's margins, which explains results (m) and (n).

Thus, while in the No Contract scenario the outside opportunities mattered for setting efforts, the Full Commitment RMI scenario solves this by having the parties sign a contract before exerting efforts. This is, however, not without cost: *w* is now "trapped" in a tradeoff, for increasing one party's margins (and thus the incentives for that party to exert efforts) implies decreasing the other's. To complicate matters more, in addition to its impact on effort *w* has also an impact on *Q*. In the next section, we will explore a mechanism that can partially mitigate this, by "liberating" *w* from having an influence on *Q*.

3.4.2 The Case with Renegotiation and Retailer Managed Inventory (RRMI)

If renegotiation was possible between $t = 1$ and $t = 2$ -*i.e.* after efforts have been exerted but before Q_{tot} is ordered-, then the following procedure would improve the manufacturer's profits:

(1) at $t = 0,$ solve the problem just as before.

(2) Between $t = 1$ and $t = 2$, offer the following new contract: set $w = c$, and extract all the retailer's profits in excess of his reservation utility via T'<0 (*i.e.*, the lump sum money transfer will be from the retailer to the supplier). Note that, because before orders are made $w = c$, $Q^*_{RRMI} = Q^*_{FB}$. Therefore, at $t = 0$ the manufacturer need not worry, when setting w**,** about Q.

This leads to the following formulation:

Manufacturer's problem

$$\underset{w,T}{Max} \quad \pi_{MANFCRMI} = \mu_1 em^*_{FCRMI}(w-c) + \mu_2 em^*_{FCRMI} + \alpha_1 er^*_{FCRMI}(w-c) - (er^*_{FCRMI})^2 - T$$

Subject to:

a) PCR

$$\int_0^{Q^*_{RRMI}} (D(r-c) + (Q^*_{RRMI} - D)(s-c))f[D]dD + \int_{Q^*_{RRMI}}^{\lambda} (Q^*_{RRMI}(r-c) + (D - Q^*_{RRMI})(g))f[D]dD$$
$$+ \mu_1 em^*_{RRMI}(r-w) + \alpha_1 er^*_{RRMI}(r-w) + \alpha_2 er^*_{RRMI} - (er^*_{RRMI})^2 + T \geq \frac{1}{4}(r-w_0)^2 \alpha_3^2$$

b) ICCM

$$em^*_{RRMI} \in \underset{em}{Argmax}\, \pi_{RRMIMAN} = \quad \mu_1 em(w-c) + \mu_2 em + \alpha_1 er(w-c) - em^2$$

c) ICCR

$$Q^*_{RRRMI}, er^*_{RRMI} \in \underset{er,Q}{Argmax}\, \pi_{RRMIRET}$$

where

$$\pi_{RRMIRET} = \int_0^{Q} (D(r-c) + (Q-D)(s-c))f[D]dD + \int_{Q}^{\lambda} (Q(r-c) + (D-Q)(g))f[D]dD$$
$$+ \mu_1 em(r-w) + \alpha_1 er(r-w) + \alpha_2 er - er^2$$

Again, since the PCR will bind,

$$
\begin{aligned}
T = \frac{1}{4}(r-w_0)^2\alpha_3^2 - (&\int_0^{Q^*_{RRMI}} (D(r-c)+(Q^*_{RRMI}-D)(s-c))f[D]dD \\
&+ \int_{Q^*_{RRMI}}^{\lambda} (Q^*_{RRMI}(r-c)+(D-Q^*_{RRMI})(g))f[D]dD \\
&+ \mu_1 em^*_{RRMI}(r-w)+\alpha_1 er^*_{RRMI}(r-w)+\alpha_2 er^*_{RRMI}-(er^*_{RRMI})^2)
\end{aligned}
$$

So, the manufacturer problem becomes:

$$
\begin{aligned}
\underset{w}{Max} \quad \pi_{MANFCRMI} = \mu_1 em^*_{FCRMI}(w-c)+\mu_2 em^*_{FCRMI} &+\alpha_1 er^*_{FCRMI}(w-c)-(er^*_{FCRMI})^2 \\
-\frac{1}{4}(r-w_0)^2\alpha_3^2 &+ \int_0^{Q^*_{RRMI}} (D(r-c)+(Q^*_{RRMI}-D)(s-c))f[D]dD \\
&+ \int_{Q^*_{RRMI}}^{\lambda} (Q^*_{RRMI}(r-c)+(D-Q^*_{RRMI})(g))f[D]dD \\
&+ \mu_1 em^*_{RRMI}(r-w)+\alpha_1 er^*_{RRMI}(r-w)+\alpha_2 er^*_{RRMI}-(er^*_{RRMI})^2
\end{aligned}
$$

Subject to:

a) ICCM

$$
em^*_{RRMI} \in \underset{em}{Argmax}\, \pi_{RRMIMAN} = \quad \mu_1 em(w-c)+\mu_2 em+\alpha_1 er(w-c)-em^2
$$

b) ICCR

$$
Q^*_{RRMI}, er^*_{RRMI} \in \underset{er,Q}{Argmax}\, \pi_{RRMIRET}
$$

where

$$
\begin{aligned}
\pi_{RRMIRET} = &\int_0^{Q}(D(r-c)+(Q-D)(s-c))f[D]dD+\int_Q^{\lambda}(Q(r-c)+(D-Q)(g))f[D]dD \\
&+\mu_1 em(r-w)+\alpha_1 er(r-w)+\alpha_2 er-er^2
\end{aligned}
$$

Because the new contract does not make the retailer worse off, (in both the new and old contracts he makes his reservation utility[vi]), he will accept this new contract. However, the manufacturer will be better off with the new contract because *Q* will rise to first best levels, thus solving part of the double marginalization problem, and efforts will increase (as we will explain later). A similar argument holds if the retailer, rather than the manufacturer offers the contract: in this case, the new contract that makes $w = c$ will include a T'', with $T < T''$, and $T'' > 0$ to compensate for the lower w. The manufacturer will accept this new contract because she will not be worse off. The retailer, on the other hand, will benefit.

It is usually assumed that renegotiation is possible if both parties benefit from discarding the old contract. Although in the situation modeled in this paper renegotiation does lead to an improvement, in reality it may still not happen for a number of reasons not modeled here (*e.g.* the time between the effort being exerted and the placement of the order for Q may not be long enough, etc.). Thus, our analysis includes both cases. However, all the insights from this paper apply regardless of whether renegotiation is allowed. From now on, if we use the sub-index RMI without specifying, we mean that the results apply to either FCRMI or RRMI.

The solutions to ICCM and ICCR are:

$$em^*_{RRMI} = \frac{1}{2}((r-w)\mu_1 + \mu_2),$$

$$er^*_{RRMI} = \frac{1}{2}((r-w)\alpha_1 + \alpha_2), \text{ and}$$

$$Q^*_{RRMI} = F^{-1}[\frac{(r-c-g)}{(r-s-g)}],$$

It is important to note that profits under RRMI are larger than under FCRMI, and that efforts differ between the two. This is due to the fact that, under FCRMI, *w* had to "coordinate" three variables: the retailer and manufacturer's efforts, and Q, while under RRMI w only affects the efforts, and thus, one restriction has been eliminated allowing higher profits.

3.4.2.2 Comparative Statics

(a) to (l) exactly as in the full commitment case (*i.e.* the signs of the partial derivatives do not change), but now:

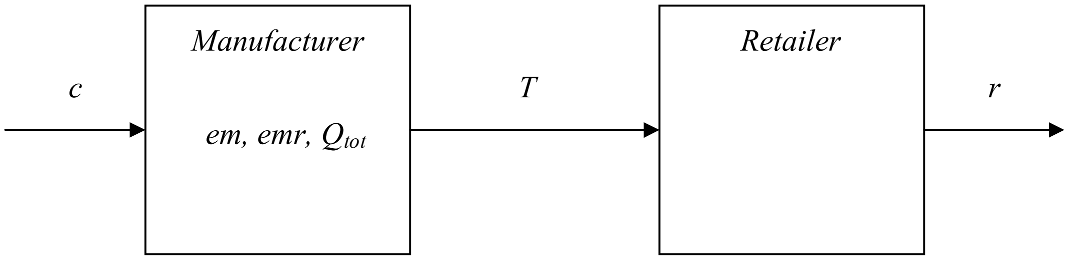
(m) $$\frac{\partial Q^*_{RRMI}}{\partial \alpha_1} = \frac{\partial Q^*_{RRMI}}{\partial \mu_1} = \frac{\partial Q^*_{RRMI}}{\partial \alpha_2} = \frac{\partial Q^*_{RRMI}}{\partial \mu_2} = \frac{\partial Q^*_{RRMI}}{\partial \alpha_3} = \frac{\partial Q^*_{RRMI}}{\partial \mu_3} = 0$$

(Proof omitted, since it would be very similar to the proof of the previous section)

Not that this is regardless of the distribution of demand. To sum up, although the inherent tradeoff in efforts that choosing *w* poses is still present, at least renegotiation can mitigate the link between *w* and Q, and achieve larger profits.

3.4.3 Then Case where the Manufacturer Takes Over

Figure *5: Manufacturer Takes Over*



In this scenario, the manufacturer takes over the retailers activities, making also stocking decisions. In exchange for this, the retailer gets paid a flat fee *T*. This can be interpreted as a "slotting allowance" in case the manufacturer only takes over one product but not the whole retailer's operation, or as a "salary" or "rent" if the manufacturer takes over all the retailer's products. Note that here, "the manufacturer takes over operations" does not necessarily mean "the manufacturer decides to buy the retailer", or "the manufacturer owns the assets of the retailer". This is a point of departure form the classical property rights theory, that is more worried about the question: "What Happens when ownership changes?" In this paper, rather, the main question is "What happens when the party in charge of an activity changes?"

The manufacturer's set of competences may be different from the retailer's, and it is possible that she will be somehow less efficient than the retailer at exerting efforts at the retailing level. This will happen, for example, in cases where a manufacturer takes control of only one product that the retailer sells, and has to place her own staff at the store, assuming that there are economies of scale in the hiring and supervising of retail employees. Let $0 \leq \beta_1 \leq 1$ represent the proportion of the manufacturer's effort at the retailer level that is directly affecting demand for Q (the lower β_1, the less the manufacturer's effort at the retail level is able to affect demand).

In addition, if the manufacturer takes over some retailing activities, it is possible that she may not benefit from some opportunities that an independent retailer has. For example, if increasing advertising at the retail level increases sales of other products that the manufacturer does not make. Let $0 \leq \beta_2 \leq 1$ represent the proportion of indirect revenues that the manufacturer can realize when she takes over the retailer's effort.

At $t=0$, the manufacturer solves the following problem:

$$\underset{T}{Max} \quad \pi_{MAN} = \int_{0}^{Q^*_{MAN}} (D(r-c)+(Q^*_{MAN}-D)(s-c))f[D]dD$$
$$+\int_{Q^*_{MAN}}^{\lambda} (Q^*_{MAN}(r-c)+(D-Q^*_{MAN})(g))f[D]dD+\mu_1 em^*_{MAN}(r-c)+\mu_2 em^*_{MAN}$$
$$+\alpha_1\beta_1 emr^*_{MAN}(r-c)+\alpha_2\beta_2 emr^*_{MAN}-(emr^*_{MAN})^2-(er^*_{MAN})^2-T$$

Subject to

$$T \geq \frac{1}{4}(r-w_0)^2\alpha_3^2 \qquad \text{(PCR)}$$

and

$$Q^*_{MAN}, em^*_{MAN}, emr^*_{MAN} \in \underset{Q,em,emr}{Argmax} \quad \pi_{MAN} \qquad \text{(ICCM)}$$

where

$$\pi_{MAN} = \int_{0}^{Q}(D(r-c)+(Q-D)(s-c))f[D]dD$$
$$+\int_{Q}^{\lambda}(Q(r-c)+(D-Q)(g))f[D]dD+\mu_1 em(r-c)+\mu_2 em$$
$$+\alpha_1\beta_1 emr(r-c)+\alpha_2\beta_2 emr-(emr)^2-(er)^2$$

Where, by *emr*, we mean, "the manufactuer's efforts at the retailer level". Again, T will be such that PCR will bind, and then the manufacturer will set efforts and quantities to maximize channel profits. Also, just as before, no trade outcomes only matter for setting T, and if the retailer, instead of the manufacturer, proposes the contract, only T will change.

A related scenario would be a more pure Vendor Managed Inventory (VMI) situation, where the manufacturer assumes decision rights over Q but does not exert effort at the retailer's level. Setting $\beta_1 = 0$, and $\beta_2 = 1$ in the above formulation, and substituting $emr^*_{MAN} = er^*_{MAN}$ -*i.e.* that what was "the manufacturer's efforts at the retailer level" is now equal to "the retailer's effort"- is mathematically equivalent to this case, because the retailer would exert effort only because of the indirect consequences of it on other products.

The solutions to the problem are:

$$em^*_{MAN} = \frac{1}{2}((r-c)\mu_1 + \mu_2) = em^*_{FB}$$

$$emr^*_{MAN} = \frac{1}{2}((r-c)\alpha_1\beta_1 + \alpha_2\beta_2)$$

and, at $t = 2$,

$$Q^*_{MAN} = Q^*_{FB} .$$

So, in this scenario, the manufacturer's efforts, and Q will be just as first best, but the efforts at the retailer level may not be.

3.4.6.1 Comparative Statics[vii]

(a) $\frac{\partial em^*_{MAN}}{\partial \mu_1} = \frac{1}{2}(r-c)$ (b) $\frac{\partial em^*_{MAN}}{\partial \mu_2} = \frac{1}{2}$ (c) $\frac{\partial em^*_{MAN}}{\partial \mu_3} = 0$

(d) $\frac{\partial em^*_{MAN}}{\partial \alpha_1} = 0$ (e) $\frac{\partial em^*_{MAN}}{\partial \alpha_2} = 0$ (f) $\frac{\partial em^*_{MAN}}{\partial \alpha_3} = 0$

(g) $\frac{\partial emr^*_{MAN}}{\partial \mu_1} = 0$ (h) $\frac{\partial emr^*_{MAN}}{\partial \mu_2} = 0$ (i) $\frac{\partial emr^*_{MAN}}{\partial \mu_3} = 0$

(j) $\frac{\partial emr^*_{MAN}}{\partial \alpha_1} = \frac{1}{2}\beta_1(r-c)$ (k) $\frac{\partial emr^*_{MAN}}{\partial \alpha_2} = \frac{1}{2}\beta_2$ (l) $\frac{\partial emr^*_{MAN}}{\partial \alpha_3} = 0$

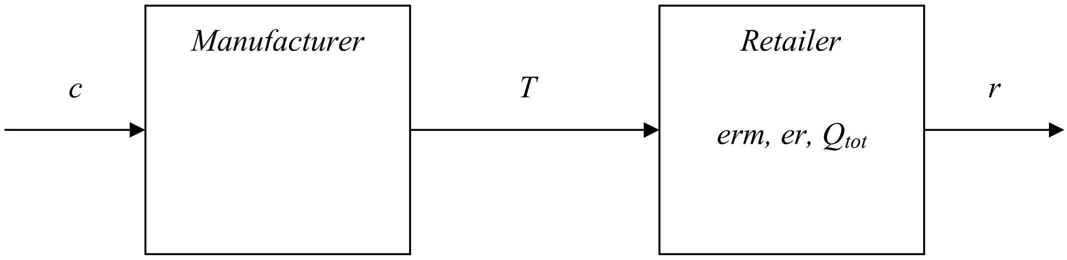
(m) $\frac{\partial Q^*_{MAN}}{\partial \alpha_1} = \frac{\partial Q^*_{MAN}}{\partial \mu_1} = \frac{\partial Q^*_{MAN}}{\partial \alpha_2} = \frac{\partial Q^*_{MAN}}{\partial \mu_2} = \frac{\partial Q^*_{MAN}}{\partial \alpha_3} = \frac{\partial Q^*_{MAN}}{\partial \mu_3} = 0$

Here, the manufacturer's efforts are exactly as in first best-results (a) to (f)-, but the manufacturer's efforts at the retail level (in the "manufacturer takes" over scenario), or the retailer's efforts (in the "pure VMI" scenario) are less than optimal –results (j) and (k). In the "manufacturer takes over" scenario, because $0 \leq \beta_2 \leq 1$ and $0 \leq \beta_1 \leq 1$,

the problem is that the manufacturer may either be less efficient –as in (j)-, or not internalize extra opportunities at the retailer level–as in (k)-. In the "pure VMI" scenario, because $\beta_1 = 0$, and $\beta_2 = 1$, the retailer exerts effort to seize extra opportunities at the retailer level just as in first best –(k) is just as first best-, but does not care for the direct effect of efforts –(j) $= 0$-.

3.4.7 The Case where the Retailer Takes Over.

Figure *6: Retailer Takes Over*



In this scenario, the retailer takes over the manufacturer's activities, making also stocking decisions. In exchange for this, the manufacturer gets paid a flat fee *T*. If the retailer takes over branding, design, supervision and other activities related to the product but not the whole manufacturer plant, it may be interpreted that *c* is the variable cost of producing the product (without any markup), and *T* is the compensation that the manufacturer gets for producing, almost like a "capacity rental", or a "cost plus" contract. If the retailer takes over the whole plant, then *T* is the "salary" or "rent" the retailer must pay the manufacturer.

Just as the case where the manufacturer takes over the retailer's operations, the retailer's set of competences may be more limited in upstream activities than the manufacturer's. For example, the independent manufacturer can be very knowledgeable about the production process in a way that allows her either to achieve lower costs or a better product if decisions where all hers (than if decisions about product materials, etc are made by a less informed party). Alternatively, there may be economies of scope in manufacturing that the retailer may not realize if it only takes over the branding and production of one product (vs the manufacturer's whole line, which may include products not sold by the retailer). Let $0 \leq \gamma_1 \leq 1$ represent the proportion of the retailer's effort at the manufacturer level that is directly affecting demand for *Q* (just as before, the lower γ_1, the less the manufacturer's retailer's effort at the manufacturer level is able to affect demand).

Similarly, if the retailer takes over the manufacturer's operation, some of the outside opportunities that an independent manufacturer could enjoy from exerting effort specific to this relationship may not be fully seized by the retailer. This may happen, for example, if channel conflicts limit the ability of the retailer to sell to other competing retailers. Let $0 \leq \gamma_2 \leq 1$

represent the proportion of indirect revenues at the manufacturer level that the retailer can realize when she takes over the manufacturer's effort.

Finally, just as before, we will model the case when the manufacturer proposes the contract (again, the case when the retailer proposes the contract yields the same efforts, quantities).

At *t=0*, the manufacturer solves the following problem:

$$\underset{T}{Max} \quad -T$$

Subject to:

$$\begin{aligned}&\int_0^{Q^*_{RET}} (D(r-c)+(Q^*_{RET}-D)(s-c))f[D]dD \\ &\quad + \int_{Q^*_{RET}}^{\lambda} (Q^*_{RET}(r-c)+(D-Q^*_{RET})(g))f[D]dD + \mu_1\gamma_1 erm^*_{RET}(r-c)+\mu_2\gamma_2 erm^*_{RET} \qquad \text{(PCR)} \\ &\qquad + \alpha_1 er^*_{RET}(r-c)+\alpha_2\beta_2 er^*_{RET} - (er^*_{RET})^2 - (erm^*_{RET})^2 + T \geq \frac{1}{4}(r-w_0)^2\alpha_3^2\end{aligned}$$

and

$$Q^*_{RET}, erm^*_{RET}, er^*_{RET} \in \underset{Q,erm,er}{Argmax} \quad \pi_{RET} \qquad \text{(ICCM)}$$

where

$$\begin{aligned}\pi_{RET} = &\int_0^{Q} (D(r-c)+(Q-D)(s-c))f[D]dD \\ &+ \int_{Q}^{\lambda} (Q(r-c)+(D-Q)(g))f[D]dD + \mu_1\gamma_1 erm(r-c)+\mu_2\gamma_2 erm \qquad \text{(ICCR)} \\ &+ \alpha_1 er(r-c)+\alpha_2 er - (er)^2 - (erm)^2\end{aligned}$$

Where, by *erm*, we mean, "the retailer's efforts at the manufacturer level". Note that, here, *T* will be negative, because the manufacturer gets some money from the retailer to compensate her for the profits she would get should she decline the contract.

The solution to the problem yields,

$$er^*_{RET} = \frac{1}{2}((r-c)\alpha_1 + \alpha_2) = er^*_{FB}$$

$$erm^*_{RET} = \frac{1}{2}((r-c)\gamma_1\mu_1 + \gamma_2\mu_2)$$

$$Q^*_{RET} = Q^*_{FB}$$

Here, Q and er are just as first best, but some efficiency may be lost at the manufacturer's level.

3.4.7.1 Comparative Statics[viii]

(a) $\frac{\partial erm^*_{RET}}{\partial \mu_1} = \frac{1}{2}(r-c)\gamma_1$ (b) $\frac{\partial erm^*_{RET}}{\partial \mu_2} = \frac{1}{2}\gamma_2$ (c) $\frac{\partial erm^*_{RET}}{\partial \mu_3} = 0$

(d) $\frac{\partial erm^*_{RET}}{\partial \alpha_1} = 0$ (e) $\frac{\partial erm^*_{RET}}{\partial \alpha_2} = 0$ (f) $\frac{\partial erm^*_{RET}}{\partial \alpha_3} = 0$

(g) $\frac{\partial er^*_{RET}}{\partial \mu_1} = 0$ (h) $\frac{\partial er^*_{RET}}{\partial \mu_2} = 0$ (i) $\frac{\partial er^*_{RET}}{\partial \mu_3} = 0$

(j) $\frac{\partial er^*_{RET}}{\partial \alpha_1} = \frac{1}{2}(r-c)$ (k) $\frac{\partial er^*_{RET}}{\partial \alpha_2} = \frac{1}{2}$ (l) $\frac{\partial er^*_{RET}}{\partial \alpha_3} = 0$

(m) $\frac{\partial Q^*_{RET}}{\partial \alpha_1} = \frac{\partial Q^*_{RET}}{\partial \mu_1} = \frac{\partial Q^*_{RET}}{\partial \alpha_2} = \frac{\partial Q^*_{RET}}{\partial \mu_2} = \frac{\partial Q^*_{RET}}{\partial \alpha_3} = \frac{\partial Q^*_{RET}}{\partial \mu_3} = 0$

This is the reciprocal of the previous case: here, the retailer's efforts are exactly as in first best-results (g) to (l)-, but the either the retailer's efforts at the manufacturer level, or the manufacturer's efforts are less than optimal –results (a) and (b). Just as before, because $0 \le \gamma_2 \le 1$ and $0 \le \gamma_1 \le 1$, the problem is that the retailer may either be less efficient –as in (a)-, or not internalize extra opportunities at the manufacturer level–as in (b)-.

4. Comparison Across Scenarios.

Observation 1

(i) $em^*_{MAN} \ge em^*_{NC}$, $em^*_{MAN} \ge em^*_{FCRMI}$, $em^*_{MAN} \ge em^*_{RRMI}$, $em^*_{MAN} \ge erm^*_{RET}$

(ii) $er^*_{RET} \geq er^*_{NC}$, $er^*_{RET} \geq er^*_{FCRMI}$, $er^*_{RET} \geq er^*_{RRMI}$, $er^*_{RET} \geq emr^*_{MAN}$

(iii) $Q^*_{FCRMI} \leq Q^*_{NC}$, $Q^*_{FCRMI} \leq Q^*_{RRMI}$, $Q^*_{FCRMI} \leq Q^*_{MAN}$, $Q^*_{FCRMI} \leq Q^*_{RET}$

Proof:

(i)

Just by noting that, when the manufacturer takes over, she exerts first best efforts.

(ii)

Similarly, noting that, when the retailer takes over, he exerts first best efforts.

(iii)

Noting that in all scenarios minus FCRMI, Q is equal to first best, while $Q^*_{FCRMI} \leq Q^*_{FB}$.

Observation 1 (i) states that when the manufacturer takes over, she exerts more upstream efforts than in any other scenario. *Observation 1* (ii) states that when the retailer takes over, he exerts more downstream efforts than in any other scenario. *Observation 1* (iii) states that Q, *i.e.* the part of Q_{total} that responds to the randomness in demand, is lower under Full Commitment Retail Manager Inventory than in any other scenario.

<u>Proposition 1</u>

For, $\mu_1(r-c)+\mu_2$ close enough to $\mu_3(w_0-c)$, and $\alpha_1(r-c)+\alpha_2$ close enough to $\alpha_3(r-w_0)$, then $\pi^*_{NC} \geq \pi^*_{RMI}$, $\pi^*_{NC} \geq \pi^*_{MAN}$,and $\pi^*_{NC} \geq \pi^*_{RET}$.

Proof:

As $\mu_1(r-c)+\mu_2 \to \mu_3(w_0-c)$, and $\alpha_1(r-c)+\alpha_2 \to \alpha_3(r-w_0)$,

$$em^*_{NC} = \frac{1}{2}(\theta((r-c)\mu_1+\mu_2)+(1-\theta)(w_0-c)\mu_3) \to \frac{1}{2}((r-c)\mu_1+\mu_2) = em^*_{FB},$$

and

$$er^*_{NC} = \frac{1}{2}((1-\theta)((r-c)\alpha_1+\alpha_2)+\theta(r-w_0)\alpha_3) \to \frac{1}{2}((r-c)\alpha_1+\alpha_2) = er^*_{FB}$$

So, $Q^*_{totalNC} \to Q^*_{totalFB}$, and, therefore, $\pi^*_{NC} \to \pi^*_{FB}$.

Proposition 1 states that if the marginal returns from trade are close enough to the marginal returns if trade does not happen, *i.e.* if the parties efforts are sufficiently not specific to the relationship, then having no contract is (weakly) the best alternative because the hold up disappears.

<u>Proposition 2</u>

For β_1 and β_2 large enough, or α_1 and α_2 small enough, $\pi^*_{MAN} \geq \pi^*_{RMI}$, $\pi^*_{MAN} \geq \pi^*_{NC}$, and $\pi^*_{MAN} \geq \pi^*_{RET}$.

Proof :

If

$\beta_1 \rightarrow 1$, and

$\beta_2 \rightarrow 1$, then

$$emr^*_{MAN} = \frac{1}{2}((r-c)\alpha_1\beta_1 + \alpha_2\beta_2) \rightarrow \frac{1}{2}((r-c)\alpha_1 + \alpha_2) = er^*_{FB}$$

Similarly, if

$\alpha_1 \rightarrow 0$, and

$\alpha_2 \rightarrow 0$, then

$emr^*_{MAN} = \frac{1}{2}((r-c)\alpha_1\beta_1 + \alpha_2\beta_2) \rightarrow 0$, and

$er^*_{FB} \rightarrow 0$.

In both cases, $Q^*_{totalMAN} \rightarrow Q^*_{totalFB}$, and, therefore, $\pi^*_{MAN} \rightarrow \pi^*_{FB}$.

Proposition 2 states that if the manufacturer can take over all the retailer's activities, perform these activities as efficiently as the retailer ($\beta_1 \rightarrow 1$), and realize all the extra benefits of the effort at the retailer level ($\beta_2 \rightarrow 1$), or if noncontractible efforts at the retail level do not influence demand significantly, ($\alpha_1 \rightarrow 0$, and $\alpha_2 \rightarrow 0$),

then it is efficient for her to either take control of the retailer's activities. Note that $\beta_2 \rightarrow 1$, if, for example, channel conflicts -both with other retailers that the manufacturer sells to or with other products that the retailer stocks- are not significant when the manufacturer moves downstream.

<u>Proposition 3</u>

For γ_1 and γ_2 large enough, or μ_1 and μ_2 small enough, $\pi^*_{RET} \geq \pi^*_{RMI}$, $\pi^*_{RET} \geq \pi^*_{NC}$, and $\pi^*_{RET} \geq \pi^*_{MAN}$.

Proof:

If

$\gamma_1 \to 1$, and

$\gamma_2 \to 1$, then

$$erm^*_{RET} = \frac{1}{2}((r-c)\gamma_1\mu_1 + \gamma_2\mu_2) \to \frac{1}{2}((r-c)\mu_1 + \mu_2) = em^*_{FB}$$

Similarly, if

$\mu_1 \to 0$, and

$\mu_2 \to 0$, then

$$erm^*_{RET} = \frac{1}{2}((r-c)\gamma_1\mu_1 + \gamma_2\mu_2) \to 0$$

and $em^*_{FB} \to 0$.

Just like Proposition 2, Proposition 3 states that if the retailer can take over all the manufacturer's activities, perform these activities as efficiently as the manufacturer ($\gamma_1 \to 1$), and realize all the extra benefits of the effort at the manufacturer level ($\gamma_2 \to 1$), or if noncontractible efforts at the manufacturer level do not influence demand significantly, ($\mu_1 \to 0$, and $\mu_2 \to 0$), then it is efficient for him to take control of the manufacturer's activities. Again, $\gamma_2 \to 1$, if, for example, channel conflicts with other retailers that the manufacturer sells to, economies of scope are not significant when the retailer takes over the manufacturer's tasks.

Propositions 1 to 3 state what would be expected in case some parameters are significantly large or small. How about intermediate cases?

4.2 A numerical example with uniformly distributed demand.

To gain insight into what would happen for intermediate values of the parameters, let us assume that *D* is uniformly distributed. Although closed form solutions can be obtained for this case, the results lack a structure that can be easily interpreted. However, a numerical example can help: let the upper bound of *D* be *100*, the lower bound be *0*, $r = 5$, $c = 2$, $w_0 = 3$, $\mu_1 = \alpha_1 = 15$, $g = s = 0$, $\mu_2 = \alpha_2 = 10$, and, finally, $\theta = \frac{1}{2}$ (that is, let us assume Nash Bargaining in the No Contract case).

Let us start by analyzing the FCRMI and RRMI cases, and then move to compare profits across all cases. In this scenario, just as section 3.4.2 predicted, RRMI efforts and quantities will differ from FCRMI, because *w* needs to coordinate one less variable. Indeed,

$$w^*_{RRMI} = \frac{c\alpha_1^2 + r\mu_1^2}{\alpha_1^2 + \mu_1^2} = \frac{7}{2} = 3.5,$$

splitting the margin exactly midway, while

$$w^*_{FCRMI} = \frac{c(2\lambda + r\alpha_1^2) + r\mu_1^2}{2\lambda + r(\alpha_1^2 + \mu_1^2)} = \frac{95}{34} \approx 2.8$$

because, under FCRMI, *w* must both induce retailer's efforts and *Q*.

This leads to:

	em^*	er^*	Q^*	Q^*_{tot}	π^*
RRMI	$\frac{65}{4} = 16.25$	$\frac{65}{4} = 16.25$	600	$\frac{2175}{2} \approx 1088$	$\frac{1725}{8} \approx 2159.4$
FCRMI	$\frac{745}{68} \approx 10.96$	$\frac{1465}{68} \approx 21.5$	$\frac{7500}{17} \approx 441$	$\frac{31575}{34} \approx 929$	$\frac{277475}{136} \approx 2040.3$
FB	$\frac{55}{2} \approx 27.5$	$\frac{55}{2} \approx 27.5$	600	1425	$\frac{4825}{2} \approx 2412.5$

The differences between RMI and the other alternatives are given by:

(a)
$$\begin{aligned}\Delta_{NC}^{FCRMI} &= \pi_{FCRMI}^{*} - \pi_{NC}^{*} = \frac{100}{17} - \frac{55}{4}\alpha_3 + \frac{1}{4}(\alpha_3)^2 - \frac{55}{8}\mu_3 + \frac{1}{16}(\mu_3)^2 \\ &\approx 6 - \frac{55}{4}\alpha_3 + \frac{1}{4}(\alpha_3)^2 - \frac{55}{8}\mu_3 + \frac{1}{16}(\mu_3)^2\end{aligned}$$

(valid for $\alpha_3 \le \frac{55}{2}$ and $\beta_3 \le 55$)

(b)
$$\begin{aligned}\Delta_{VMI}^{FCRMI} &= \pi_{FCRMI}^{*} - \pi_{VMI}^{*} = \frac{52225}{136} - \frac{2025}{4}(\beta_1)^2 - 225\beta_1\beta_2 - 25(\beta_2)^2 \\ &\approx 384 - \frac{2025}{4}(\beta_1)^2 - 225\beta_1\beta_2 - 25(\beta_2)^2\end{aligned}$$

(c)
$$\begin{aligned}\Delta_{RET}^{FCRMI} &= \pi_{FCRMI}^{*} - \pi_{RET}^{*} = \frac{52225}{136} - \frac{2025}{4}(\gamma_1)^2 - 225\gamma_1\gamma_2 - 25(\gamma_2)^2 \\ &\approx 384 - \frac{2025}{4}(\gamma_1)^2 - 225\gamma_1\gamma_2 - 25(\gamma_2)^2\end{aligned}$$

(d) $\Delta_{NC}^{RRMI} = \pi_{RRMI}^{*} - \pi_{NC}^{*} = 125 - \frac{55}{4}\alpha_3 + \frac{1}{4}(\alpha_3)^2 - \frac{55}{8}\mu_3 + \frac{1}{16}(\mu_3)^2, \alpha_3 \le \frac{55}{2}$ and $\beta_3 \le 55$

(e)
$$\begin{aligned}\Delta_{VMI}^{RRMI} &= \pi_{RRMI}^{*} - \pi_{VMI}^{*} = \frac{4025}{8} - \frac{2025}{4}(\beta_1)^2 - 225\beta_1\beta_2 - 25(\beta_2)^2 \\ &\approx 503 - \frac{2025}{4}(\beta_1)^2 - 225\beta_1\beta_2 - 25(\beta_2)^2\end{aligned}$$

(f)
$$\begin{aligned}\Delta_{RET}^{RRMI} &= \pi_{RRMI}^{*} - \pi_{RET}^{*} = \frac{4025}{8} - \frac{2025}{4}(\gamma_1)^2 - 225\gamma_1\gamma_2 - 25(\gamma_2)^2 \\ &\approx 503 - \frac{2025}{4}(\gamma_1)^2 - 225\gamma_1\gamma_2 - 25(\gamma_2)^2\end{aligned}$$

The results can be summarized as follows: either in the full commitment, or in the renegotiation case, if $\gamma_1, \gamma_2, \beta_1, \beta_2, \mu_3$ and α_3 are small enough, then $\pi_{RMI}^{*} > \pi_{NC}^{*}, \pi_{RMI}^{*} > \pi_{MAN}^{*}$, and $\pi_{RMI}^{*} > \pi_{RET}^{*}$. That is, if the losses when the retailer takes over are high enough, the losses when the manufacturer takes over are high enough, and the outside opportunities are low enough (or, equivalently, effort specificity is high enough), then either FCRMI or RRMI will maximize profits.

5. Conclusion

In this paper, we study the full specter of possibilities that two parties within a supply chain face: from not signing any contract at all before transacting, passing through signing a contract but keeping each party doing different tasks, and finally arriving to one party taking control of the other party's activities. Thus, this is a first attempt to study supply chain contracting issues in a broader framework. In addition, we introduce some of the complexities and tradeoffs involved when both manufacturer and retailer exert uncontractible effort. We attempt to answer the following questions: what happens when a party becomes responsible for an activity? In the presence of non-contractible effort, when is no contract better than a contract? Our intention is not to be exhaustive, rather, we hope that this is just one of many paper to come about both topics.

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Appendix

Proof of 3.4.1.1

Lemma 1

$$\frac{dw^*_{FCRMI}}{d\alpha_1} < 0$$

Proof:

$$\frac{\partial^2 \pi_{FCRMI}}{\partial \alpha_1 \partial w} = (r - c)\frac{\partial er^*_{FCRMI}}{\partial w}$$

but

$$\frac{\partial^2 ICCR_{FCRMI}}{\partial er \partial w} = -\alpha_1 < 0$$

so

$$\frac{\partial er^*_{FCRMI}}{\partial w} < 0$$

Lemma 2

$$\frac{dw^*_{FCRMI}}{d\mu_1} > 0$$

Proof:

$$\frac{\partial^2 \pi_{FCRMI}}{\partial \mu_1 \partial w} = (r - c)\frac{\partial em^*_{FCRMI}}{\partial w}$$

but

$$\frac{\partial^2 ICCM_{FCRMI}}{\partial em \partial w} = \mu_1 > 0$$

so

$$\frac{\partial^2 er^*_{FCRMI}}{\partial \mu_1} = \mu_1 > 0$$

Using Lemmas 1 and 2, and calculating the crosspartial derivatives of the corresponding ICC, it can be shown that:

(a) $\frac{\partial em^*_{FCRMI}}{\partial \mu_1} > 0$ (d) $\frac{\partial em^*_{FCRMI}}{\partial \alpha_1} < 0$ (g) $\frac{\partial er^*_{FCRMI}}{\partial \mu_1} < 0$

(j) $\frac{\partial er^*_{FCRMI}}{\partial \alpha_1} > 0$ (m) $\frac{\partial Q^*_{FCRMI}}{\partial \mu_1} < 0$ (n) $\frac{\partial Q^*_{FCRMI}}{\partial \alpha_1} > 0$

Also, calculating the crosspartial derivatives of the corresponding ICC yields,

(k) $\frac{\partial er^*_{FCRMI}}{\partial \alpha_2} = \frac{1}{2}$ (l) $\frac{\partial er^*_{FCRMI}}{\partial \alpha_3} = 0$

(b) $\frac{\partial em^*_{FCRMI}}{\partial \mu_2} = \frac{1}{2}$ (c) $\frac{\partial em^*_{FCRMI}}{\partial \mu_3} = 0$

(e) $\frac{\partial em^*_{FCRMI}}{\partial \alpha_2} = 0$ (f) $\frac{\partial em^*_{FCRMI}}{\partial \alpha_3} = 0$

(h) $\frac{\partial er^*_{FCRMI}}{\partial \mu_2} = 0$ (i) $\frac{\partial er^*_{FCRMI}}{\partial \mu_3} = 0$

Notes

[i] That the parties may not be able to contract on quantities is the most common (if sometimes implicit) assumption in the supply chain literature. There are a number of reasons why quantities on the shelves may not be contractible, but the main one is, perhaps, the difficulty in verifying how many units are actually there (vs in the retailer's backroom, warehouse, diverted to other retailers, etc.).

[ii] This assumption is made purely for to simplify calculations. Insights could be generalized to any generic effort cost function $V(e)$, with $V(0) = 0$, $V'(.) \geq 0$ and $V''(.) > 0$.

[iii] Note that setting $w = c$ makes the manufacturer vulnerable to diversion by the retailer, who could buy from the manufacturer at cost and sell to other retailers. This is, however, not an exclusive problem of the no contract scenario. As it will become clear later, the renegotiated contracts scenario is also in the same situation, and the full commitment contract scenario may also be.

[iv] The proofs are omitted because results are a simple derivation from the optimal efforts and quantity.

[v] The proofs of this case are in the appendix. Not that the derivation of these comparative statics is more involved than the other cases because we were unable to find general closed form solution for efforts, and quantity. Thus, we have to resort to finding the crosspartial derivatives of the corresponding function.

[vi] The model can be generalized to assume that, at renegotiation, each party gets a fraction of the gains from renegotiation. This does not alter any of the paper's insights, its only effect being that renegotiation becomes "less powerful" in terms of its distance to first best: indeed, in the extreme case where the retailer gets all the gains from trade, renegotiation would be equal to full commitment.

[vii] The proofs are omitted because results are a simple derivation from the optimal efforts and quantity.

[viii] The proofs are omitted because results are a simple derivation from the optimal efforts and quantity.

NOTAS

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